Quantum neural networks

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References

1. MVA, N.E.Kaputkina, V.A.Krylov. Quantum neural networks: Current status and prospects for development. *Physics of Particles and Nuclei* **45**(2014) 1013-1032
2. MVA, N.N. Zolnikova, N.E. Kaputkina, V.A. Krylov, Yu. E. Lozovik and N.S. Dattani. Towards a feasible implementation of quantum neural networks using quantum dots. *Appl. Phys. Lett.* **108**(2016)103108
3. MVA, N.N. Zolnikova, N.E. Kaputkina, V.A. Krylov, Yu. E. Lozovik and N.S. Dattani. Decoherence and Entanglement Simulation in a Model of Quantum Neural Network Based on Quantum Dots. *EPJ Conf.* **108**(2016)02006
4. MVA, N.N. Zolnikova, N.E. Kaputkina, V.A. Krylov, Yu. E. Lozovik and N.S. Dattani. Entanglement in a quantum neural network based on quantum dots. *PNFA* **24**(2017)24-28
5. MVA, N.E.Kaputkina, V.A.Krylov. Symmetry and decoherence-free subspaces in quantum neural networks, arXiv.org:1802.05710, to appear in Russ. J. Nucl. Phys.

Subjects

- Where QNN come from?

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- Quantum simulators, cellular automata, self-organization

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- Quantum Hopfield network optical implementation
- Quantum neural nets on quantum dots
- Simulation of QNN at finite temperature

Historical notes

It is widely believed that the first paper on quantum neural networks was

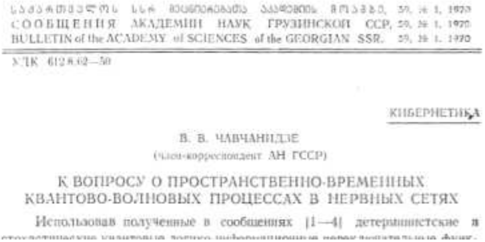
S. Kal, 1995

S.Kak. On quantum neural computing. *Inf. Sci.* **83**(1995)143-160,

However

V. Chavchanidze, 1970

On spatial-temporal quantum-wave processes in neural networks. *Soobshch. AN Gruzinskoi SSR* **59**(1)(1970)37-40 [in Russian]


СООБЩЕНИЯ АКАДЕМИИ НАУК ГРУЗИНСКОЙ ССР, 59, № 1, 1970
BULLETIN of the ACADEMY of SCIENCES of the GEORGIAN SSR. 59, № 1. 1970

УДК 612.8.02—50

КИБЕРНЕТИКА

В. В. ЧАВЧАНИДЗЕ
(член-корреспондент АН ГССР)

К ВОПРОСУ О ПРОСТРАНСТВЕННО-ВРЕМЕННЫХ КВАНТОВО-ВОЛНОВЫХ ПРОЦЕССАХ В НЕРВНЫХ СЕТЯХ

Использовав полученные в сообщениях [1—4] детерминистские и

L.Behera, I.Kar, A.Elitzur. A recurent quantum neural network model to describe eye tracking of moving targets. *Found. Phys. Lett.* **18** (2005) 357:

A target movement can be simulated by brain using the wave-packet described by Schrödinger equation

$$\imath\hbar\frac{\partial\Psi(t,x)}{\partial t} = -\frac{\hbar^2}{2m}\Delta\Psi(t,x) + V(t,x)\Psi(t,x)$$

with adjustable potential $V(t,x) = \sum_i W_i(t,x)\exp((\nu(t) - g_i)^2)$. The squared modulus of the solution gives the probability to find the (image of) target in certain domain: $f(t,x) = |\Psi(t,x)|^2$.

This idea was latter generalized to multi-agent games:

V.G. Ivanceic and D.J. Reid. Dynamics of confined crowds modelled using Entropic Stochastic Resonance and Quantum Neural Networks. *Int. J. Intell. Def. Supp. Sys.* **2**(2009)269

$$\imath\frac{\partial\psi_i(t,x)}{\partial t} = -D[\psi]\frac{\partial^2\psi_i}{\partial x^2} + VU_i[\psi]\psi_i(t,x)$$

where $D[\psi]$ is nonlinear diffusion coefficient, and $U_i[\psi] = |\psi_i(t,x)|^2$ is the p.d.f. of the i-th type agent.

$$V(t,x,\omega) = \sum_{i=1}^{n}\omega_i g_i(x)$$

self-learning potential Learning rule:

$$\dot{\omega}_i = -\omega_i + c_H \max_{x,k\neq l}|\psi_k(t,x)|g_i(x)|\psi_l(t,x)|$$

Local basic potentials are:

$$g_i(x) = \frac{1}{\sqrt{2\pi}\sigma_i}e^{-\frac{(x-\bar{x}_i)^2}{2\sigma_i^2}}, \quad \bar{x}_i = \frac{\int_M \bar{\psi}_i(t,x)x\psi_i(t,x)dx}{\int_M \bar{\psi}_i(t,x)\psi_i(t,x)dx}.$$

System with two types of agents

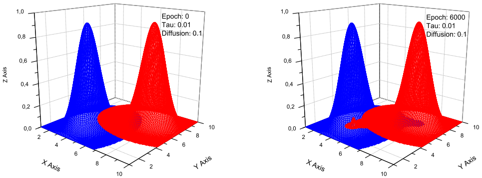
$$
\begin{aligned}
\imath \frac{\partial \psi_B(t,x)}{\partial t} &= -\frac{a_B}{2}|\psi_R|^2 \frac{\partial^2 \psi_B}{\partial x^2} \\
&+ V|\psi_B|^2 \psi_B(t,x), \\
\imath \frac{\partial \psi_R(t,x)}{\partial t} &= -\frac{a_R}{2}|\psi_B|^2 \frac{\partial^2 \psi_R}{\partial x^2} \\
&+ V|\psi_R|^2 \psi_R(t,x), \\
\dot{\omega}_i &= -\omega_i \\
&+ c_H \max_x |\psi_R| g_i |\psi_B|, \\
i &= B, R
\end{aligned}
$$

128×128 grid

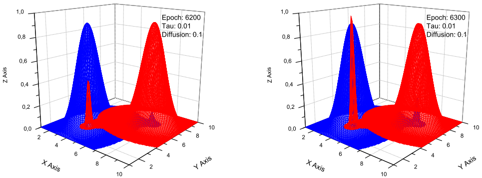
$(\sigma_x, \sigma_y) = (0.38, 0.50)$,

$a_B = a_R = 0.1, c_H = 0.05$

$T = 0, 0.6$



$T = 0.62, 0.63$



Pictures from:

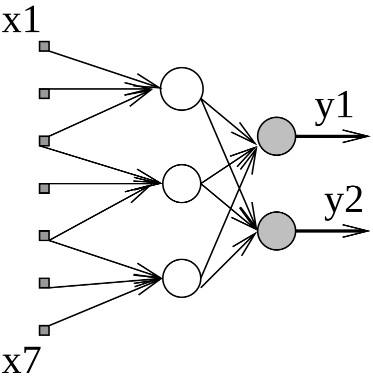
M.Altaisky, N.Kaputkina, V.Krylov. *Phys. Part. Nuclei.* **45**(2014)1013

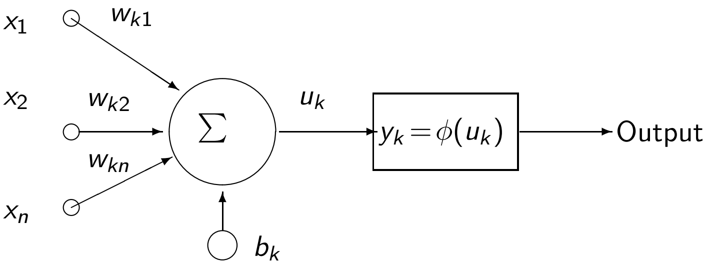
What is neural network?

NN [S.Haykin.Neural Networks,Pearson Education:1999]:

A neural network is a massively parallel distributed processor made up of simple processing units, which has a natural propensity for storing experimental knowledge and making it available for use. It ressembles the brain in two respects:

1. Knowledge is acquired by the network from its environment through a learning process
2. Interneuron connection strengths, known as synaptic weights, are used to store acquired knowledge





$$u_k = \sum_{j=1}^{N} w_{kj} x_j + b_k, \qquad y_k = \phi(u_k)$$

Choice of sigmoid function $\phi(u) =$

$$\theta(u), \qquad tanh(u), \qquad \frac{1}{1+e^{-u}}$$

Classical artificial neural networks (CANN)

Types of learning

$(w_{ij}(t+1) = w_{ij}(t) + \Delta w_{ij})$

- Supervised learning
- Unsupervised learning

Types of neural networks

- Feed-forward networks
  Multilayer perceptron
- Recurrent networks
- Self-Organized networks
  Hopfield networks
- Fuzzy networks

Hebb rule

$$\Delta w_{ij} = \eta y_j y_i, 0 < \eta \leq 1$$

$$\Delta w_{ij} = \eta y_j d_i, etc.$$

Cost function

$$E = \frac{1}{2}\sum_{i,k}\left(d_i^{(k)} - y_i^{(k)}\right)^2,$$

i – neuron; k – sample vector

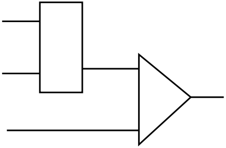
Classical information

Bit

Physical system that may be in either of distinct physical states "0" or "1","off" or "on",↓ or ↑

$$0+0=0, 0+1=1, \quad 1+1=10$$
$$0*0=0, 0*1=0, \qquad 1*1=1$$
$$\neg 0=1 \qquad \neg 1=0$$

Logical circuits



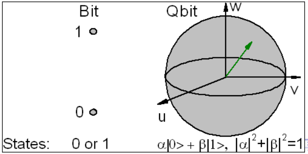
Quantum information

Qubit =quantum bit

$$|\psi\rangle = c_0|0\rangle + c_1|1\rangle,$$
$$c_0, c_1 \in \mathbb{C}, \quad |c_0|^2 + |c_1|^2 = 1$$

Bloch sphere

$$|\theta,\phi\rangle = \quad \cos\frac{\theta}{2}|\uparrow\rangle + e^{\imath\phi}\sin\frac{\theta}{2}|\downarrow\rangle,$$
$$0 \le \theta \le \pi; 0 \le \phi \le 2\pi.$$



Qubit network

- Nonlinearity can be introduced by measuring procedure R.Zhou et al. in Proc ICANN 2006.
- The nonlinearity is introduced by using the phase of the complex amplitude u Kouda, Matsui, Nishimura, Peper. 2005:

$$y = \frac{\pi}{2}\frac{1}{1+e^{-\sigma}} - \arctan\frac{\Im(u)}{\Re(u)}, \quad u = \sum_{k=1}^{n} e^{\imath\frac{\pi}{2}x_k} e^{\imath\theta_k} - e^{\imath\lambda}$$

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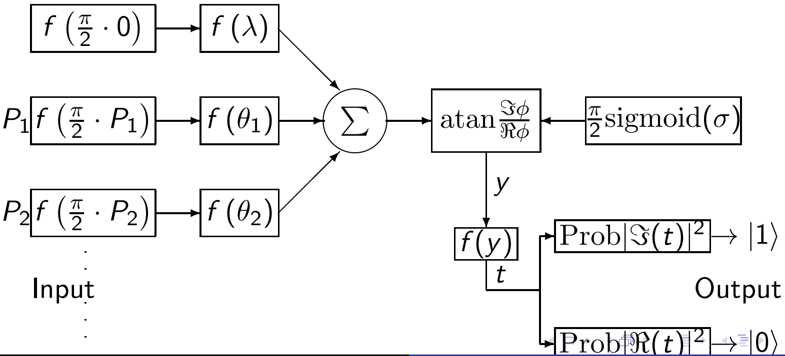
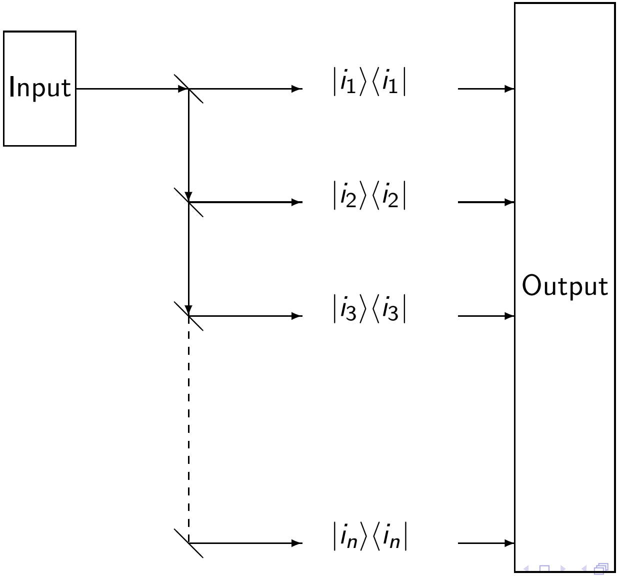


Let us have to store $N = k \times m$ size image, represented by an array of N Ising spins $\xi_i = \pm 1$. Each image corresponds to the matrix J_{ij}, so that the Hamiltonian

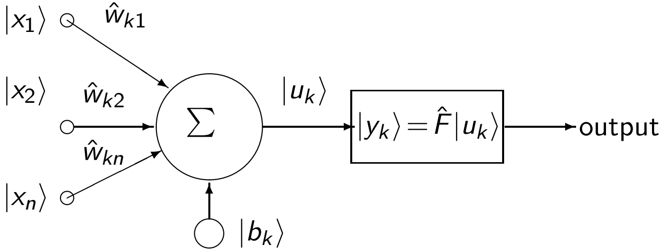
$$H = -\frac{1}{2}\sum_{ij} J_{ij}\xi_i\xi_j + \sum_j b_j\xi_j,$$

has minimum for this image. The system with the Hamiltonian H is Hopfield network J.J.Hopfield *PNAS* **79**(1982)2554 In case the sigmoid function is the sgn function, the network $y_i(t) = \mathrm{sgn}(\sum_{i\neq j} J_{ij}y_j(t-1))$ is stationary if $J_{ij} = \frac{1}{N}\xi_i\xi_j$. If we want to store p different images $J_{ij} = \frac{1}{N}\sum_{l=1}^{p}\xi_i^{(l)}\xi_j^{(l)}$.
For an unknown image $\xi^{(new)}$ the amplitude $\langle\xi^{(new)}|\xi^{(l)}\rangle$ has the maximal value for the basic (l-th) image closest to unknown.
In quantum case connection matrix is a projector: $\hat{J} = \sum_{i=1}^{n} |i\rangle\langle i|$

Associative memory – image classification
A.Vlasov, arxiv.org:quant-ph/9703010



Quantum perceptron (M.V.Altaisky arxiv.org/quant-ph/0107012)



$$|u_k\rangle = \sum_{j=1}^{N} \hat{w}_{kj}|x_j\rangle, \quad |y_k\rangle = \hat{F}|u_k\rangle.$$

At the absence of interaction with environment, $\hat{F}$ should be a *linear*.
Learning rule ($F = 1$): $\hat{w}_j(t+1) = \hat{w}_j(t) + \eta\left(|d\rangle - |y(t)\rangle\right)\langle x_j|$

$$\left||d\rangle - |y(t+1)\rangle\right|^2 = \left||d\rangle - \sum_{j=1}^{n} \hat{w}_j(t+1)|x_j\rangle\right|^2 = (1-n\eta)^2 \left||d\rangle - |y(t)\rangle\right|^2$$

Quantum neural networks vs. quantum computational networks

Lack of unitarity

The problem of linking quantum neurons together:
How to implement the response function ?
Who and how measures the neuron state ?
How we can cope permanent interaction with environment?

Links to real brain networks

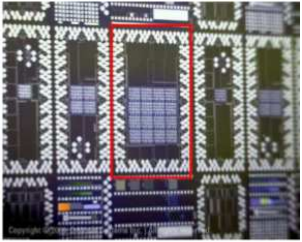
Quantum tunneling in brain neurons definitely takes place at room temperature:
F.Beck and J.Eccles, *PNAS* **89**(1992)11357

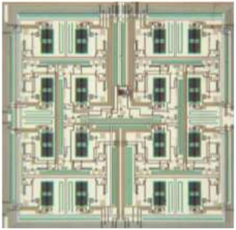
Quantum annealing machines, D-wave Systems Inc.
M.Johnson et al. *Nature* **473**(2011)194

The first scalable quantum computer was constructed by D-Wave Systems Inc. It is capable of solving exponentially difficult minimization problems

$$H_P = -\sum_{i=1}^{N} h_i \sigma_i^z + \sum_{i,j=1}^{n} J_{ij} \sigma_i^z \sigma_j^z$$

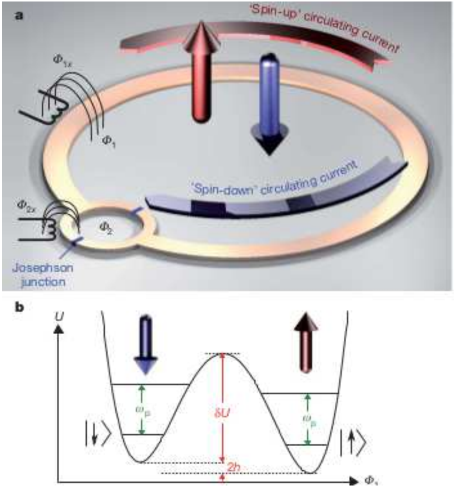
in polynomial time. The 'spins' σ_i^z are implemented in Superconducting Quantum Interference Devices. The connection matrix J_{ij} is implemented by inductive couplings. The superconducting circuit technology is used.





128 qubit SQUID processor.
From arxiv.org:1204.2821

SQUID flux qubit



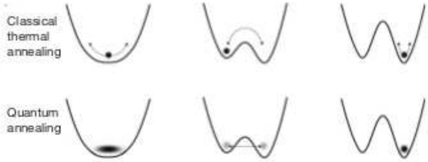
Redrawn from M.Johnson et al.
Nature **473**(2011)194

Realization of quantum annealing

- All 'spins' are initialized in X direction at $t = 0$

$$H(t) = -\Gamma(t)\sum_{i=1}^{n}\Delta_i(t)\sigma_i^x + \Lambda(t)\left[-\sum_{i=1}^{n}h_i\sigma_i^z + \sum_{i,j=1}^{n}J_{ij}\sigma_i^z\sigma_j^z\right]$$

- The transverse magnetic field is adiabatically turned off $\Gamma(t) \to 0$ with simultaneous increase of $\Lambda(t) \to 1$



Redrawn from M.Johnson et al. *Nature* **473**(2011)194

Optical implementation of Hopfield network

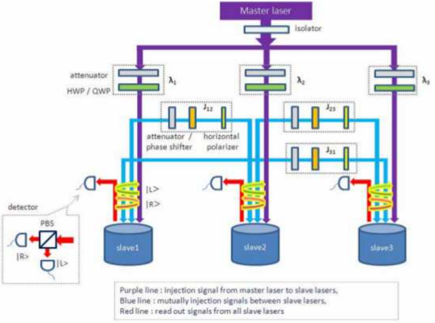
Optical implementation of N-spin Ising model

$$H = -\sum_{1\le i<j\le N} J_{ij}\sigma_i\sigma_j$$

http://qnncloud.com

- T. Inagaki et al. Large-scale Ising spin network based on degenerate optical oscillators. *Nature Photonics* **10**(2016)415
- P.L.McMahon et al. A fully programmable 100-spin coherent Ising machine with all-to-all connections. *Science* **354**(2016)614
- H. Takesue et al. Quantum neural network for solving complex combinatorial optimization problems. *NTT Technical Review* **15**(7)(2017)

One master laser and M mutually injection locked slave lasers. Ising model is implemented by coherent feedback network using optical interference circuits instead of measurements.



A spin σ_{iz} is represented by right or left polarization state of each slave laser

$$H = \sum_{i<j} J_{ij}\sigma_{iz}\sigma_{jz} + \sum_i \lambda_i \sigma_{iz}$$

Output readout

$$\sigma_{iz} = \begin{cases} +1 & n_{R_i} > n_{L_i} \\ -1 & n_{R_i} < n_{L_i} \end{cases}$$

Fig. 1. A proposed injection-locked laser system for finding the ground state of an Ising model Eq. (2). A master laser output is equally split into M paths and injected into M slave lasers via an optical isolator. At a time $t < 0$ (initialization), the injection signal from the master laser has a vertical linear polarization so that all slave lasers are initialized in vertical linear polarization states $|V\rangle_1|V\rangle_2 \ldots |V\rangle_M$. At a time $t = 0$, the combined attenuator, HWP and QWP can implement the Zeeman term λ_i. Also at a time $t = 0$, each slave laser output is injected to other slave lasers via a horizontal linear polarizer, phase shifter, and attenuator but without an isolator. This mutual injection-locking can implement the Ising interaction term J_{ij}. After a steady state condition is reached, the two polarization components of each slave laser are detected by a polarization beam splitter (PBS) and two photodetectors.

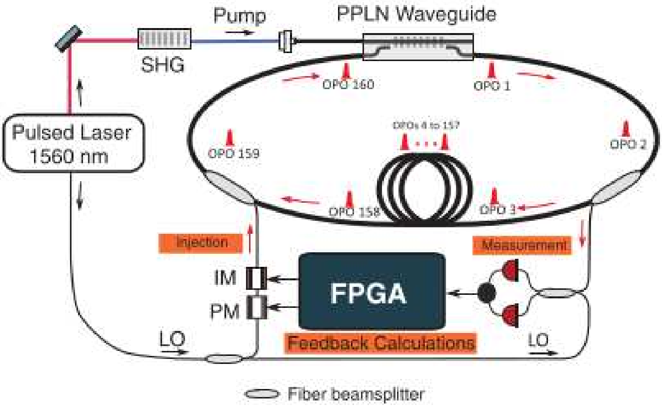


Fig. 1. Experimental schematic of a measurement-feedback–based coherent Ising machine. A time-division–multiplexed pulsed degenerate optical parametric oscillator is formed by a nonlinear crystal [periodically poled lithium niobate (PPLN)] in a fiber ring cavity containing 160 pulses. A fraction of each pulse is measured and used to compute a feedback signal that effectively couples the otherwise-independent pulses in the cavity. IM, intensity modulator; PM, phase modulator; LO, local oscillator; SHG, second-harmonic generation; FPGA, field-programmable gate array.

ARTICLES NATURE PHOTONICS DOI: 10.1038/NPHOTON.2016.68

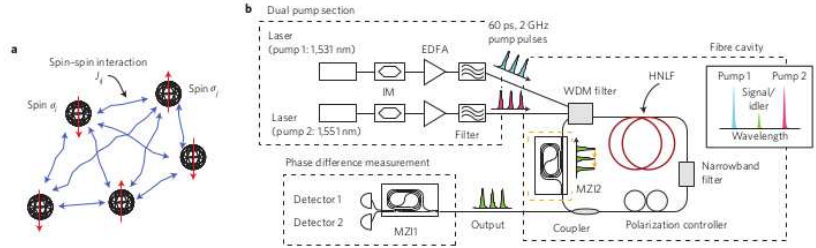
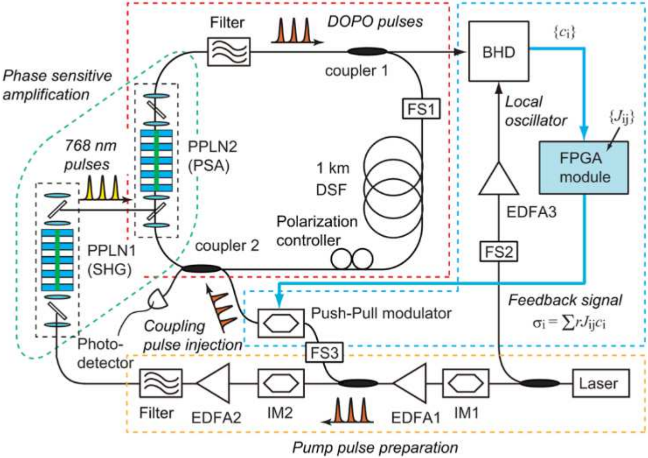
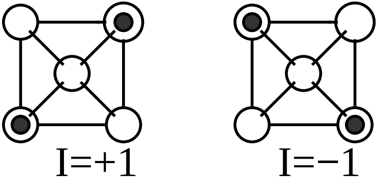


Figure 1 | Ising model and set-up for generating artificial Ising spins based on DOPOs. a, An Ising model. **b**, Experimental set-up. IM, intensity modulator; EDFA, erbium-doped fibre amplifier; WDM, wavelength division multiplexing; HNLF, highly nonlinear fibre; MZI, delayed Mach–Zehnder interferometer. The difference in the propagation times of the two arms of the MZIs is 500 ps for both MZI1 and MZI2. An EDFA and an optical bandpass filter (omitted for clarity) were placed in front of MZI1 to pre-amplify the DOPO signal and suppress noise from the EDFA, respectively. Inset: Wavelength allocation of pumps 1 and 2 and the signal/idler wave. MZI2 is inserted when simulating the 1D Ising model.

A time-division-multiplexed pulsed degenerate optical parametric oscillator is formed by a nonlinear crystal [periodically poled lithium niobate (PPLN) in a fiber ring cavity]



In 2000 E.C.Behrman et al. proposed a model of quantum neural network where the nonlinearity is attained by means of interaction of quantum dot array with the substrate phonons. Each neuron was designed as a "molecule" of 5 quantum dots



The evolution of the N neuron array is given by

$$|\psi_{1\ldots N}(T)\rangle = \sum_{traject} e^{\frac{\imath}{\hbar}\sum_j\left[K_i\sigma_x^i(j\Delta t)+E_i(j\Delta t)\sigma_z^i(j\Delta t)\right]} I[\vec\sigma_z(t)]|\psi_{1\ldots N}(0)\rangle,$$

$$I[\vec\sigma_z(t)] = \int\prod_k D[x_k(t)] e^{\frac{\imath}{\hbar}\int_0^T d\tau \sum_k\left[\frac{m_k\dot x_k^2}{2}+\frac{m_k\omega_k^2x_k^2(\tau)}{2}+\lambda_k^i x_k(\tau)\sigma_z^i(\tau)\right]}$$

$$H = \sum_{i=1}^{2} \frac{\Delta_i}{2}\sigma_z^{(i)} + \sum_{i=1}^{2} \frac{K_i}{2}\cos(\omega_L t)\sigma_x^{(i)} + \sum_{i\neq j} J_{ij}\sigma_+^{(i)}\sigma_-^{(j)}$$
$$+ \sum_{a,i} g_a x_a |X_i\rangle\langle X_i| + H_{phonon},$$

Δ_i is the energy gap between the ground and the first excited state of the i-th QD; K_i is a coupling to an external driving field, J_{ij} is the dipole-dipole coupling, constructed in analogy to the dipole-dipole interaction of atoms.

$$\sigma_z^{(i)} = |\mathrm{X}_i\rangle\langle\mathrm{X}_i| - |0_i\rangle\langle 0_i|, \sigma_x^{(i)} = |0_i\rangle\langle\mathrm{X}_i| + |\mathrm{X}_i\rangle\langle 0_i|,$$
$$\sigma_+^{(i)} = |\mathrm{X}_i\rangle\langle 0_i|, \sigma_-^{(i)} = |0_i\rangle\langle\mathrm{X}_i|.$$

The phonon modes x_a are assumed to interact only to the excited states $|X_i\rangle$

Simple model

We consider a pair of identical QDs

$$\Delta_1 = \Delta_2 = \Delta, J_{12} = J_{21} = J, K_1 = K_2 = K$$

in which we can see that in the limit of vanishing driving field ($K \to 0$) the eigenstates of H_{Ex} are

$$\frac{|\mathrm{X}0\rangle - |0\mathrm{X}\rangle}{\sqrt{2}}, \quad \frac{|\mathrm{X}0\rangle + |0\mathrm{X}\rangle}{\sqrt{2}}, \quad |00\rangle, \quad |\mathrm{XX}\rangle$$

corresponding to the eigenvalues $(-J, J, -\Delta, \Delta)$. The first two states have zero eigenvalue with respect to the interaction with phonons $V = \sum_{\alpha,i} g_\alpha x_\alpha |\mathrm{X}_i\rangle\langle\mathrm{X}_i|$, and thus survive in coherent superposition even in the presence of a bath of acoustic phonons.

Phonon bath parametrization

The free phonon Hamiltonian is $H_{Ph} = \sum_a \frac{p_a^2}{2m_a} + \frac{m_a\omega_a^2 x_a^2}{2}$.
The phonons in GaAs substrate are assumed to have the spectral density

$$J(\omega) = \frac{\pi}{2}\sum_a \frac{g_a}{m_a\omega_a}\delta(\omega-\omega_a) \approx \alpha\omega^3 \exp(-(\omega/\omega_c)^2),$$

which defines the bath correlation function

$$R(t) = \int_0^\infty \frac{d\omega}{\pi} J(\omega)\left[\cos(\omega t)\coth\left(\frac{\omega}{2k_B T}\right) - \mathrm{i}\sin(\omega \mathrm{t})\right].$$

This form for $J(\omega)$ is the excellent agreement between experiment and theory of the interaction of single quantum dot with phonon bath [A.J.Ramsay et al., *PRL* **104**(2010)017402, **105**(2010)177402; D.P.S. McCutcheon et al, *PRB* **84**(2011)081305R; N.S.Dattani, *CPC* **184**(2013)2828]

$$\alpha = \frac{(D_e - D_h)^2}{4\pi^2\rho\hbar v_s^5} = 0.027\mathrm{ps}^2 \text{for GaAs}, \quad \omega_c = \frac{\sqrt{2}v_s}{d} = 2.2\mathrm{ps}^{-1}$$

Functional integral solution of von Neumann equation

Makarov and Makri *J. Chem. Phys.* **102** (1995) 4600

In our studies we used the quasi-adiabatic propagator path integral (QUAPI) technique for the solution of the von Neumann equation for the density matrix $\rho(t)$, which describes the evolution of the above described pair of interacting quantum dots:

$$\dot{\rho} = \mathrm{tr}_{\mathrm{Ph}} \left(-\frac{\imath}{\hbar} [H, \rho_{\mathrm{tot}}] \right),$$

with the initial condition:

$$\rho_{\mathrm{tot}}(0) = \rho(0) \otimes \frac{e^{-\beta H_{\mathrm{Ph}}}}{\mathrm{tr}\left(e^{-\beta H_{\mathrm{Ph}}}\right)}.$$

where for the particular case of two interacting QDs

$$\rho(0) = |\psi(0)\rangle\langle\psi(0)| \ , \ |\psi(0)\rangle = \frac{1}{\sqrt{2}} \left(|0X\rangle + |X0\rangle \right)$$

Quasiadiabatic path integral

The time dependence of the reduced density matrix of the QD system is given by the Feynman integral

$$\langle s_N^+|\rho(t)|s_N^-\rangle = \int \left(\prod_{m=0}^{N-1} \langle s_{m+1}^+|e^{-\frac{\imath\Delta t}{\hbar}H_{OQS}}|s_m^+\rangle\langle s_m^-|e^{\frac{\imath\Delta t}{\hbar}H_{OQS}}|s_{m+1}^-\rangle\right) \times$$

$$\times \qquad \langle s_0^+|\rho(0)|s_0^-\rangle I\left(\{s_m^\pm\}^N;\Delta t\right)\prod_{m=0}^{N-1} ds_m^+ ds_m^-,$$

where s_m^+ (s_m^-) denotes the state of the OQS at $t_m = m\Delta t$ on the time-forward (time-backward) propagation. The discretized bath influence functional is equal to

$$I\left(\{s_m^\pm\}^N;\Delta t\right) = e^{-\sum_{mm'}(s_m^+ - s_m^-)(\eta_{mm'}s_{m'}^+ - \eta_{mm'}^* s_{m'}^-)},$$

with $\eta_{mm'}$ being the discretized version of correlator given in N.S.Dattani, *CPC* 184 (2013)2828 according to quasi-adiabatic propagator path integral method of Makarov and Makri.

Numerical codes for path integrals
M.V.Altaisky et al. *EPJ WoC* **108**(2016)02006

A F77 and C++ codes were designed for evaluation of density matrices of systems interacting with heat bath according to the method described in A.Vagov, M.Croitoru et al., *PRB* **83** (2011)094303. The total evolution time t is divided into N time slices:

$$\rho_{\alpha_N,\beta_N} = e^{it\left(\hat{\Omega}_{\beta_N\beta_N}-\hat{\Omega}_{\alpha_N\alpha_N}\right)} \sum_{\{\alpha_n,\beta_n\}} \prod_{n=1}^{N} M_{\alpha_n}^{\alpha_{n-1}} M_{\beta_{n-1}}^{\beta_n *} \prod_{n'=1}^{n} e^{S_{nn'}} \rho_{\alpha_0\beta_0},$$

where $\hat{\Omega} = \mathrm{diag}\,(0,\Delta,\Delta,2\Delta)$ is the diagonal part of the system Hamiltonian without bath, $\alpha,\beta \in \{00, 0X, X0, XX\}$.
The evaluation is performed using the method of augmented density matrix evaluation (Makarov and Makri, 1995): $\overline{R}_n = T_n\overline{R}_{n-1}$, where $\overline{R}_n$ coincides with the density matrix of the system R_n for all discrete time instants less or equal to the memory length n_c, or is truncated by the last time instant $n - n_c - 1$.
The results were also cross tested with the FeynDyn code N.S.Dattani, *CPC* **184**(2013)2828

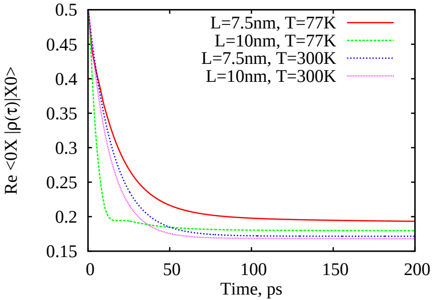
Quantum dot parameters

Ramsay et al. *Phys. Rev. Lett.* **105**(2010)177402;
McCutcheon et al. *Phys. Rev. B* **84**(2011)081305;
Dattani N.S. *Comp. Phys. Comm.* **184**(2013)2828

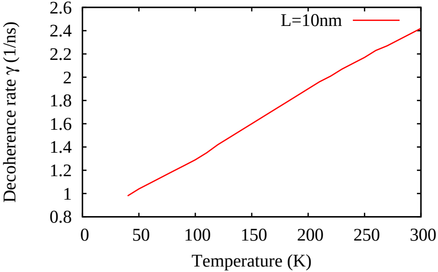
$$m_* = 0.067 m_e, \rho_{GaAs} = 5.37 g/cm^3, v_s = 5.11 \cdot 10^5 cm/s, \\ \epsilon = 10.0, a_0 = 3.94 nm, E_0 = 36.5 meV$$

$d = 3.3$nm, $L = 10$nm
Dipole coupling $J = 0.595 ps^{-1}$
Driving field $K = 0.476 ps^{-1}$
Energy gap
$\Delta = 158 ps^{-1} \approx 104 meV$
Cutoff frequency $\omega_c = 2.2 ps^{-1}$

d=33nm, L = 50nm
Dipole coupling $J = 0.476 ps^{-1}$
Driving field $K = 4.76 ps^{-1}$
Energy gap
$\Delta = 1.58 ps^{-1} \approx 1.04 meV$
Cutoff frequency $\omega_c = 0.22 ps^{-1}$



Re[<0X|ρ(t)|X0>] was fitted to exp(-γt)



Parameters of the model

$d = 3.3\mathrm{nm}$,

$\mu = 79.3\mathrm{Debye}$

$K = 0.24\mathrm{ps}^{-1}$

$\approx 950V/cm$

$J = \frac{\mu^2}{\varepsilon L^3}$

$J \approx 1.4\mathrm{ps}^{-1}$

for $L = 7.5$nm

Initial state

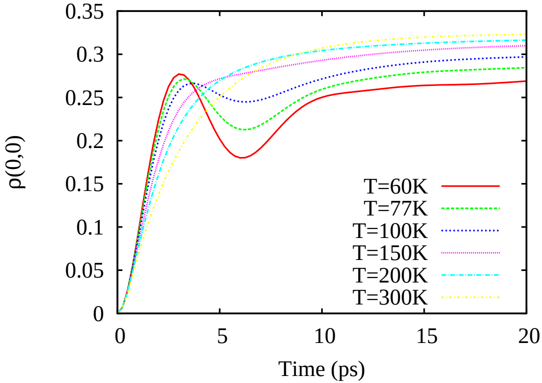
$|\psi(0)\rangle = \frac{|0X\rangle + |X0\rangle}{\sqrt{2}}$

Memory:

$n_c = 5$

Ground state density evolution

M.V.Altaisky et al. PNFA **24**(2017)24 (arXiv.org:1512.01141)



Parameters

$L = 10$ nm

Four eigenvalues $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \lambda_4$ of the auxiliary matrix

$$R(\rho) = \sqrt{\sqrt{\rho}\rho^*\sqrt{\rho}},$$

where ρ^* denotes the complex conjugation, are used to evaluate the *concurrence* $C = \max(0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4)$. The entanglement of formation is then given by

$$E(\rho) = H\left(\frac{1}{2} + \frac{1}{2}\sqrt{1 - C^2}\right),$$

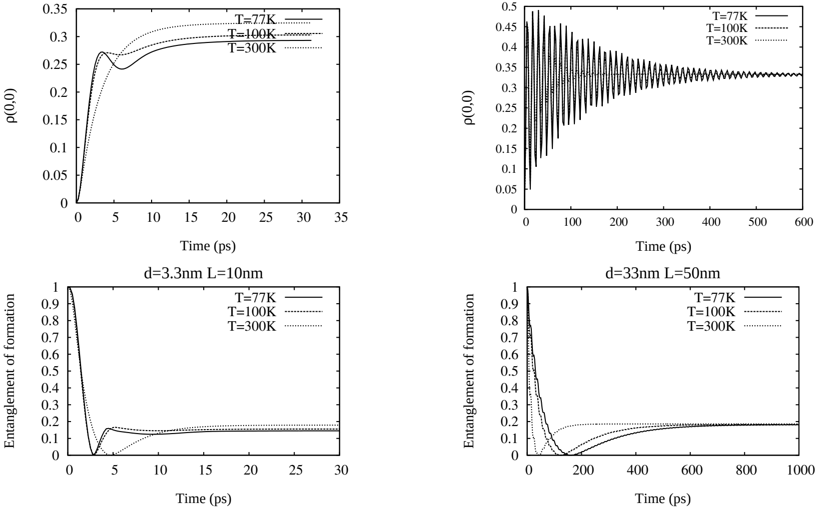
where
$H(x) = -x\log_2 x - (1-x)\log_2(1-x)$,
is a binary entropy function.

The entanglement of the singlet state is exactly one.

Bell basis

$$|e_1\rangle = \frac{1}{\sqrt{2}}(|XX\rangle + |00\rangle)$$
$$|e_2\rangle = \frac{\imath}{\sqrt{2}}(|XX\rangle - |00\rangle)$$
$$|e_3\rangle = \frac{\imath}{\sqrt{2}}(|X0\rangle + |0X\rangle)$$
$$|e_4\rangle = \frac{1}{\sqrt{2}}(|X0\rangle - |0X\rangle)$$

Evolution of Entanglement [Altaisky et al, EPJ WoC 108(2016)02006]



The asymptotic value of the density matrix ρ written in magic basis is $\rho(+\infty) = \mathrm{diag}(1/3, 1/3, 1/3, 0)$.

THANK YOU FOR YOUR ATTENTION !!!

Perspectives

- Low energy consuming QNN
- Compatibility with optical devices