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Cryogenic traveling-wave parametric amplifier as possible broadband source of microwave biphotons

A. B. Zorin, J. Felgner, C. Kißling, M. Khabipov and R. Dolata

2018-2021

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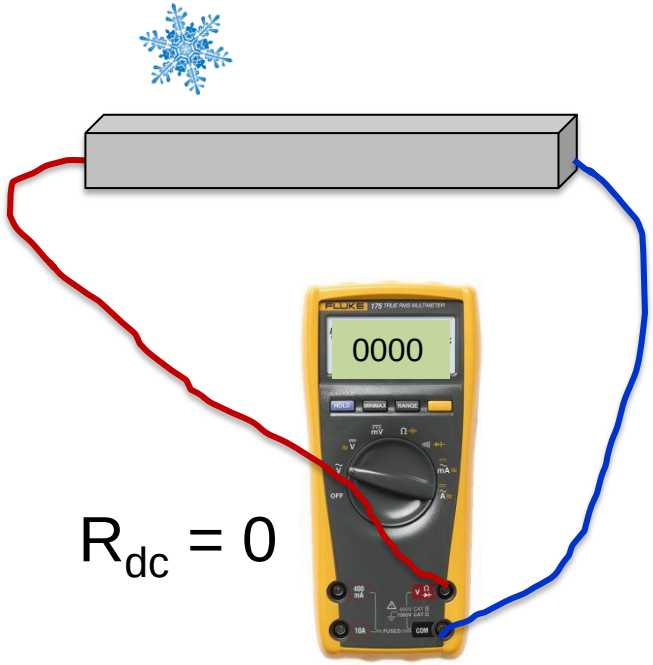
Content

- Traveling-wave parametric amplifier (TWPA) based on superconducting technology
  - (a) Concept
  - (b) Experiment at PTB
- TWPA as a broadband source of nonclassical microwaves
  - (a) Amplification of quantum vacuum
  - (b) SPDC vs. dynamical Casimir effect

Why **superconducting technology?**

At **T < T_c** it enables lossless dc/low-frequency wiring on chip



$R_{dc} = 0$

For example,

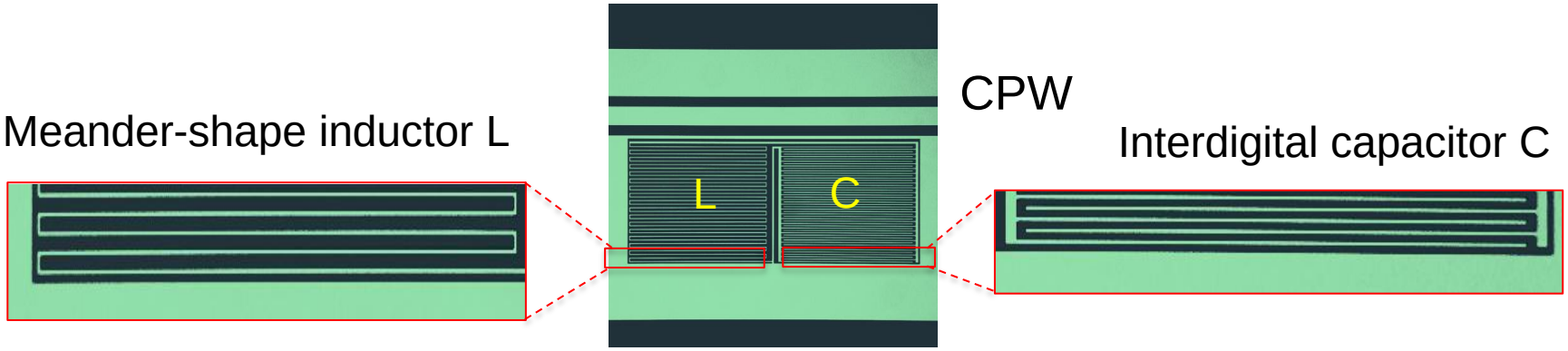
critical temperature of

Al –	$T_c \approx 1.2$ K
Nb –	$T_c \approx 9.3$ K
V –	$T_c \approx 5.4$ K
NbN –	$T_c \approx 16$ K

Why **superconducting technology?**

It enables microwave circuits (e.g. high-Q resonators) with very low damping

(for $f \ll \Delta_{supercond.}/h \approx 1.76\, k_B T_c/h \sim$ 50-300 GHz)



Example: LC-resonator (10 GHz) coupled to a coplanar transmission line (CPW)

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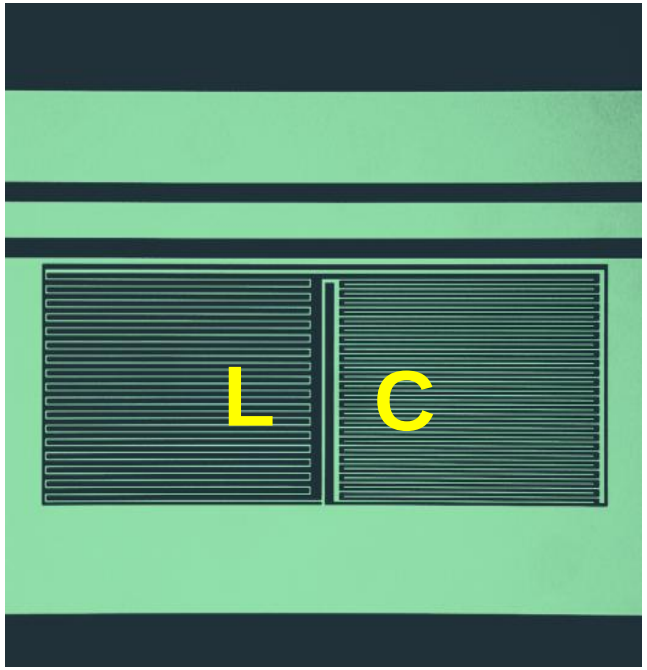
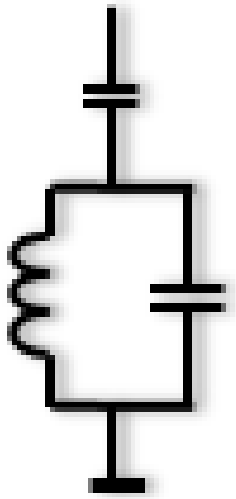




$$\omega = \frac{1}{\sqrt{LC}}$$

Can operate in quantum regime:

$$\frac{\hbar\omega}{2} \approx k_B \times 240\ \mathrm{mK} \quad (\text{for } 10\,\mathrm{GHz})$$

Sufficiently low temperature is needed!

Cryogenic technique for millikelvin temperatures

Dilution refrigerator:

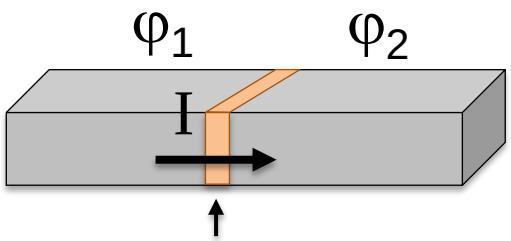
Typically, base temperature **5–20 mK**, cooling power 100–500 μW

Why **superconducting technology?**

- Enables **nonlinear** and **magnetically-controlled** circuit elements

Josephson effect (1962):

Superconducting tunnel junction



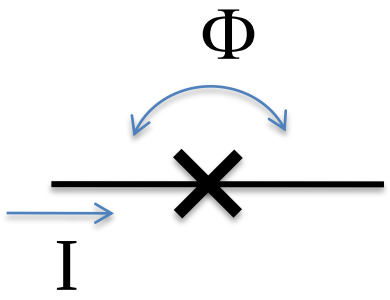
(thin insulator, ~ **1-2 nm**)

Josephson junction (JJ)
- symbolic notation

Supercurrent $\boxed{I = I_c \sin(\varphi_1 - \varphi_2) = I_c \sin\varphi}$, I_c is critical current

At $|I| \leq I_c$, current I flows without dissipation (voltage V = 0)!

Josephson **phase difference** φ $\Leftrightarrow$ magnetic flux $\Phi = \frac{\Phi_0}{2\pi}\varphi \quad \propto \int V\,dt$



where $\Phi_0 = h/2e \approx 2.07\times10^{-15}$ Wb – the flux quantum

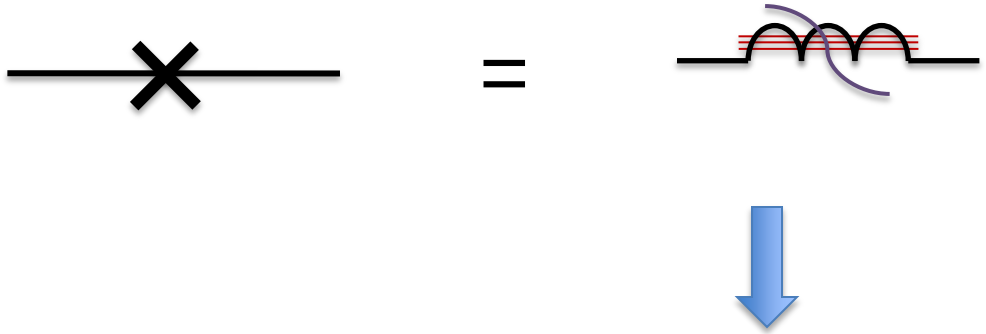
$$I = I_c \sin\varphi = I_c \sin\left(2\pi\frac{\Phi}{\Phi_0}\right) = \boxed{\frac{2\pi I_c}{\Phi_0}\left(1-\frac{\Phi^2}{6}+\ldots\right)}\Phi$$

$\leftarrow$ formula like $I \approx L^{-1}\Phi$

This term can be interpreted as inverse (Josephson) inductance L_J^{-1}

+ nonlinear terms!

Josephson junction (JJ)

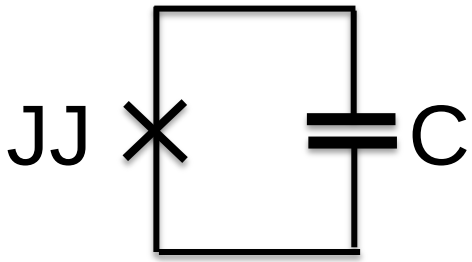


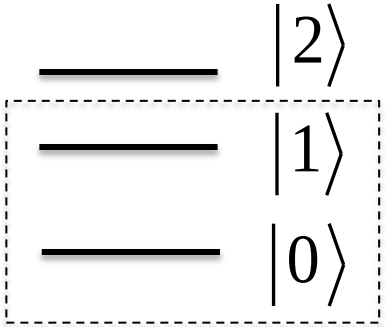
Nonlinear inductor

$$I(\Phi) = L_0^{-1}\left[\Phi - \gamma\,\Phi^3\right]$$

Kerr-like term

Anharmonic quantum oscillators
(e.g., **transmon** qubits)





Qubit basis

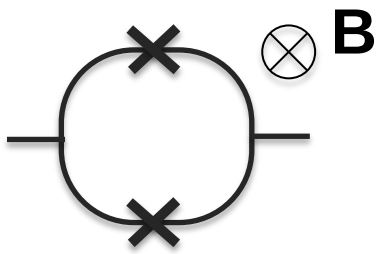
Josephson junction (JJ)

=

Nonlinear inductor

$$I(\Phi) = L_0^{-1}\left[\Phi - \gamma\,\Phi^3\right]$$

Two-junction interferometer (SQUID)



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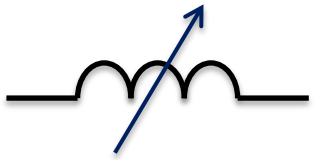
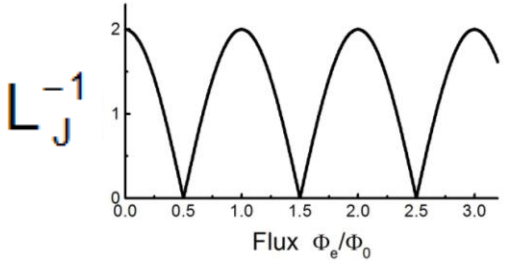


Magnetically-controlled inductor (for $I \ll I_c$)

Interference of 2 Josephson currents



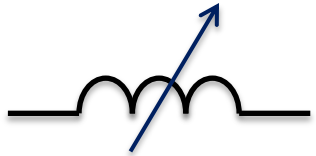
$\Phi_e = BS$

Nonlinear inductor

(for $I \sim I_c$)

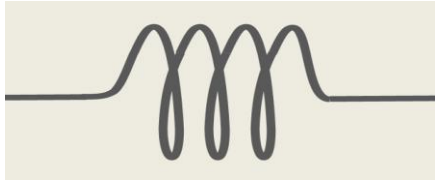
$I(t)$

Magnetically-controlled inductor (for $I \ll I_c$)



$\mathbf{B}(t)$

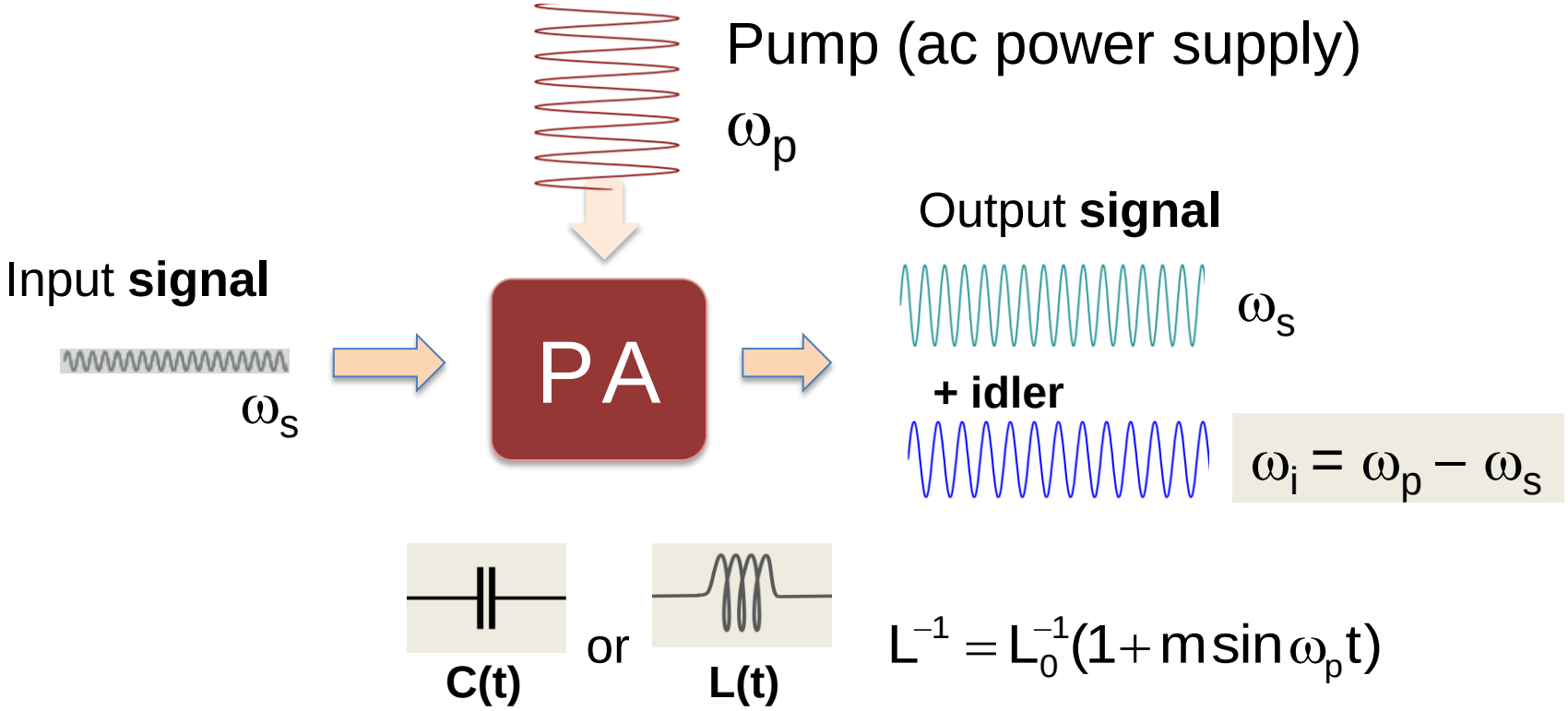
$L(t)$



Time-dependent parameter

The core element of **parametric** oscillators and amplifiers

Parametric Amplifier



Based on reactive elements, periodically (with frequency ω_p) varied in time.

Ideally, **parametric amplifiers** do not add noise to the signal!

Cryogenic PA, including **Josephson Parametric Amplifiers (JPA)**, can have

$$T_{noise} \sim \frac{\hbar\omega}{2k_B} \leq 0.5\,K$$ (non-degenerate, i.e. phase-insensitive JPA)

JPA with **quantum-limited performance** are necessary in many fields!

These JPAs are already available (f ~ 5-20 GHz), but their bandwidth (typically, ~ 10-50 MHz) is sometimes insufficient!

Motivation

Josephson Parametric Amplifiers (JPAs) with quantum-limited performance and **large bandwidth** are urgently needed!

- Integration with **quantum sensors** (SQUID, SET, nanomechanical oscillator, em-/particle-detectors, …)

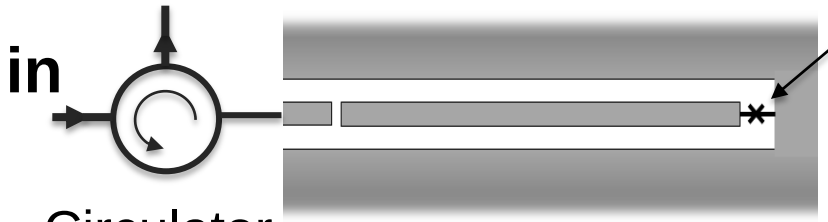
- **QI applications** (quantum communication, quantum computing, …)

Conventional architecture

Reflection-type

Large pump
+ small signal

$\omega_p \approx \omega_s$



Superconducting
cavity

Q » 1

Josephson Junction
(Kerr inductance)

$$L_J(t) = \left(1 + \alpha I^2(t)\right) L_0$$

or
direct pumping (of SQUID)
$L_J(t)$

Large **Q** → effective mode mixing, but **limited bandwidth**!

→ Gain – bandwith trade-off

A broad-band JPA should be **free of cavity**!

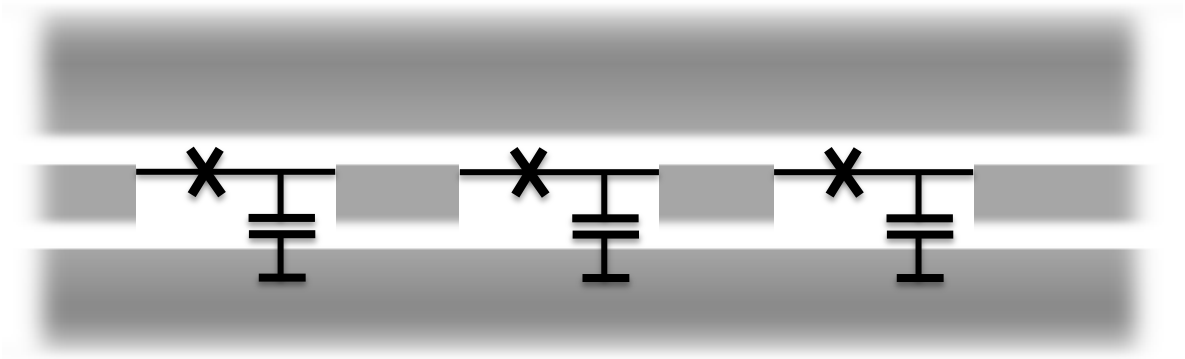
Traveling-wave JPA (TWJPA) architecture

Last 5 years – big progress…

Traveling-wave JPA (TWJPA): basic idea

CPW line with embedded Josephson junctions (N ~ 1000)

Yaakobi et al. PRB **87**, 144301 (2013)



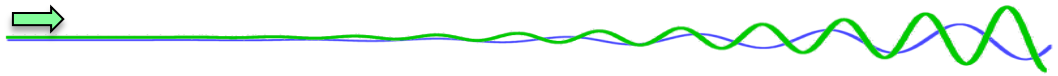
Mixing of traveling microwaves

Pump

Pump (ω_p)

$$2\omega_p = \omega_s + \omega_i$$

Signal



Amplified signal (ω_s)
+ Idler (ω_i)

Ideally, exponential growth of signal!

Principle of operation

Optical-fiber parametric amplifier (OPA) is based on **Kerr nonlinearity**,

$$\boldsymbol{P} = \chi^{(1)}\,\boldsymbol{E} + \chi^{(3)}\,\boldsymbol{EEE} + \ldots$$

Enables four-wave mixing!

$$2\omega_p = \omega_s + \omega_i$$

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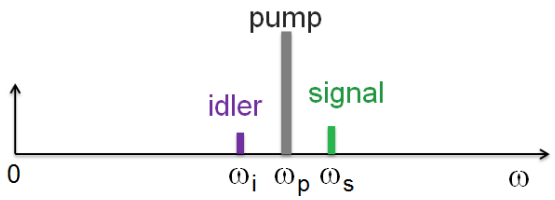
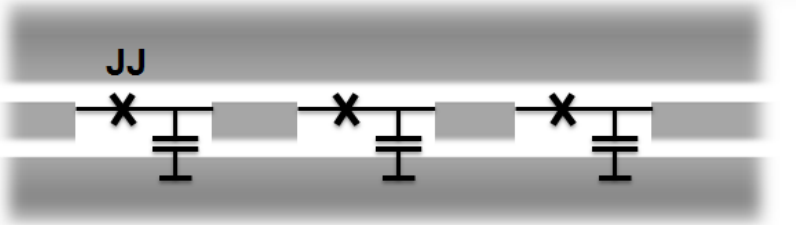
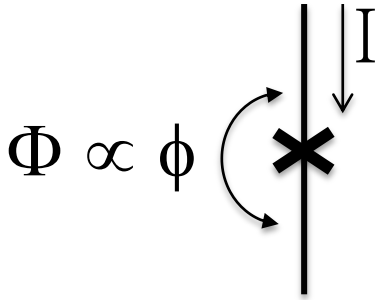


Josephson-junction transmission line (TWJPA):



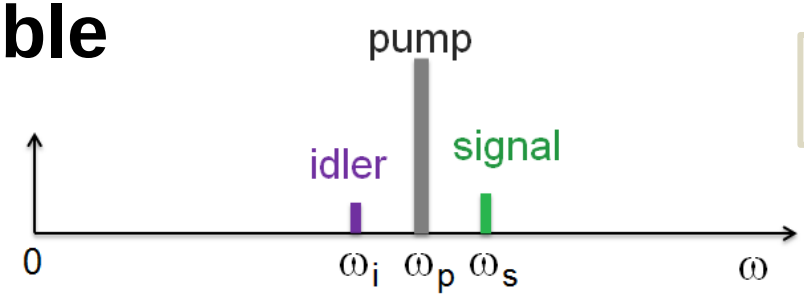


$$I = I_c \sin\phi = \frac{2\pi I_c}{\Phi_0}\left(\Phi - \tfrac{1}{6}\Phi^3 + \ldots\right)$$

← **Kerr-like nonlinearity**

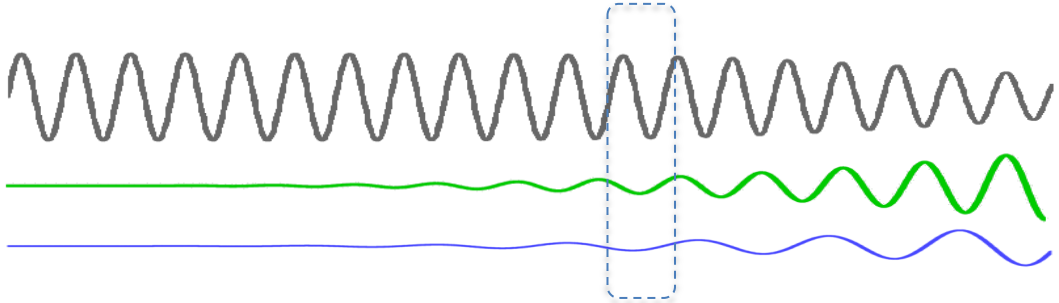
Traveling-wave JPA (TWJPA)

4-wave mixing is possible



$$2\omega_p = \omega_s + \omega_i$$

Problem: phase matching

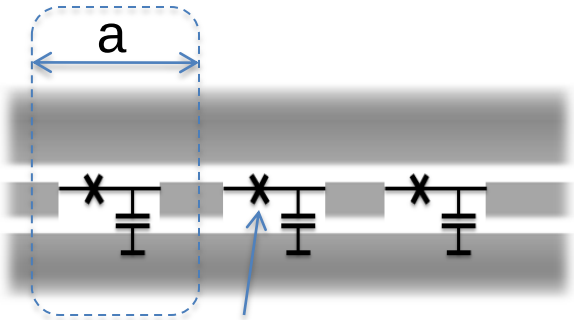


$$2k_p = k_s + k_i$$

$\forall$ x, the “correct” relation between phases must be satisfied!

Why the relation $2k_p = k_s + k_i$ is not fulfilled automatically?

1. **Chromatic dispersion** of the line



Made of lumped elements,
$\rightarrow$ cutoff $\omega_0 = (LC)^{-1/2}$

Self-capacitance of junctions C_J,
Josephson plasma $\omega_{pl} = (LC_J)^{-1/2}$

$$k = \frac{2}{a}\arcsin\frac{\omega/2\omega_0}{\sqrt{1-(\omega/\omega_J)^2}} \approx \frac{\omega}{a\omega_0}\left(1+\frac{\omega^2}{2\omega_J^2}+\frac{\omega^2}{24\omega_0^2}\right)$$

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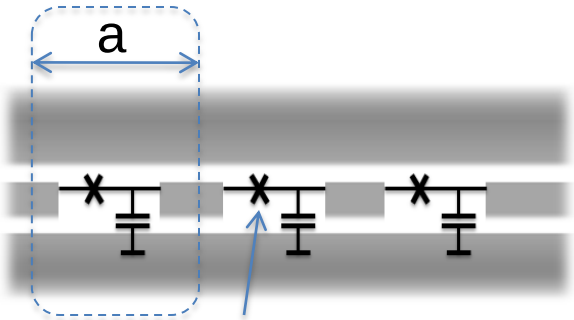

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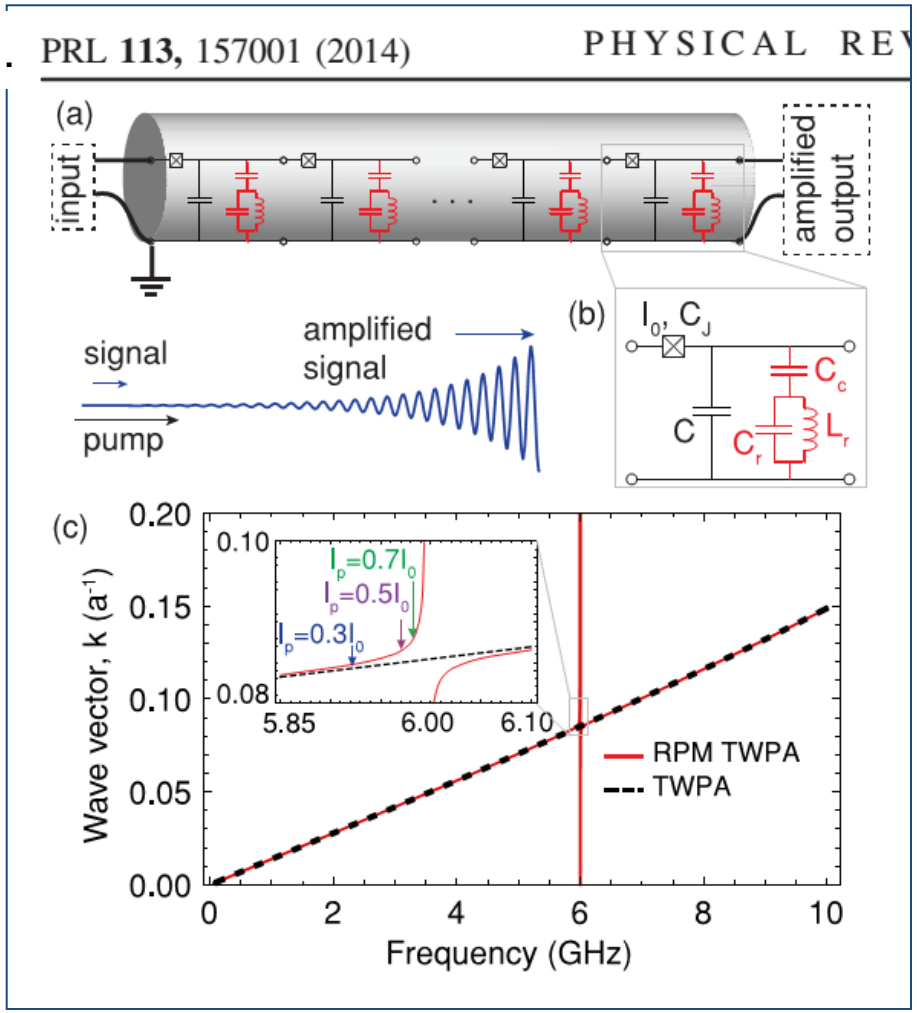
2. **Phase modulation** due to Kerr nonlinearity $I(\Phi) = L_0^{-1}\left[\Phi - \gamma\,\Phi^3\right]$

k depends on signal power and should be compensated!

TWJPA: phase matching problem

K. O'Brien et al.



Dispersion engineering

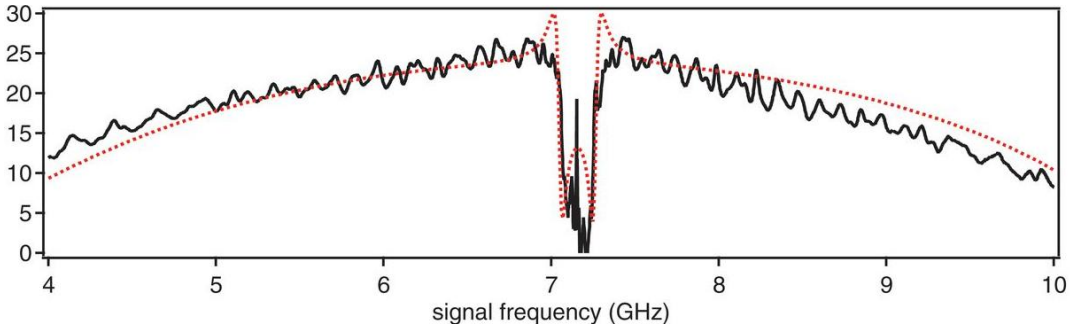
$\Rightarrow$ Modified $k(\omega)$ relation in vicinity of ω_{res}

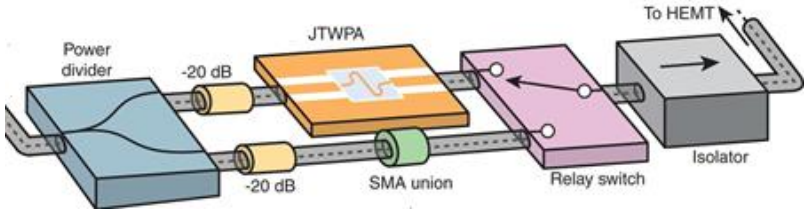
Good phase matching in a reasonable bandwidth

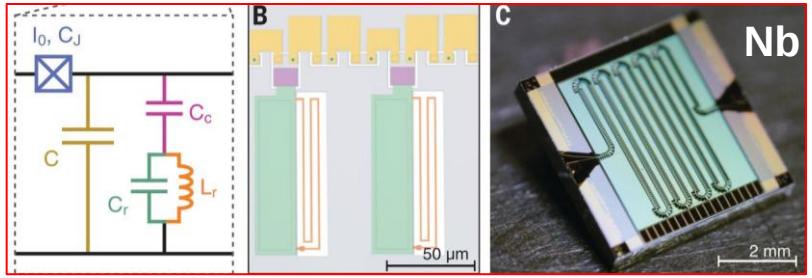
$$k_s + k_i \approx 2k_p$$

Available TWJPAs exploiting dispersion engineering

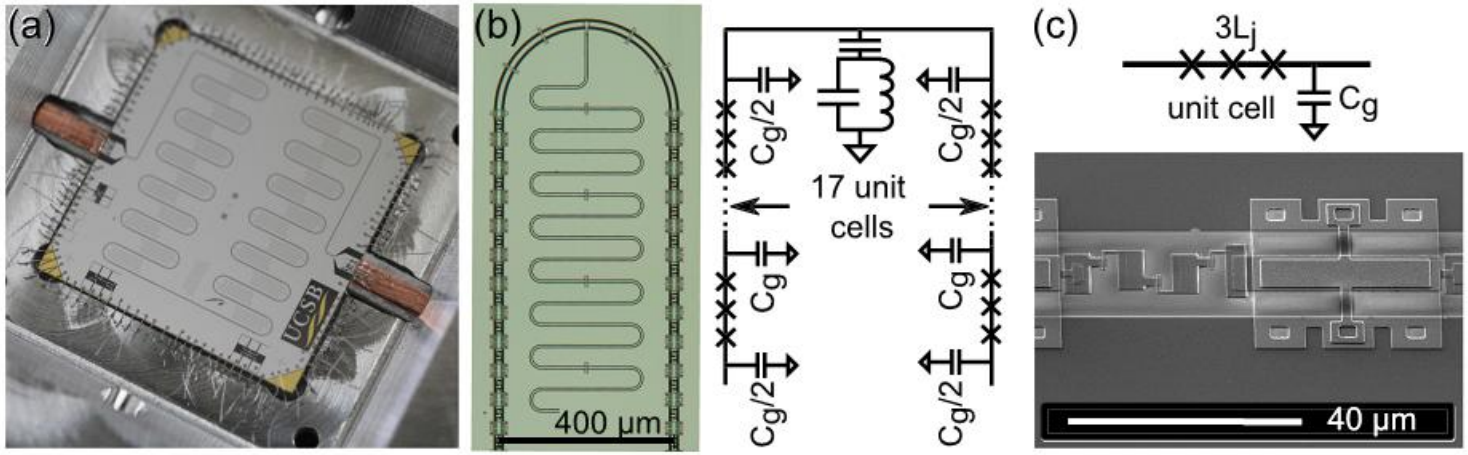
Macklin et al. Science **350**, 307 (2015) – **UC Berkeley**





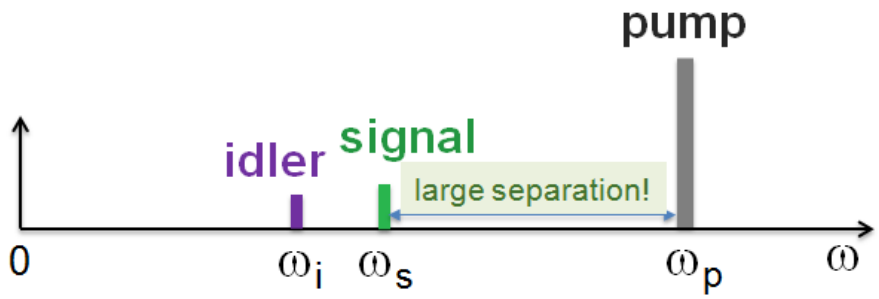


White et al. APL **106**, 242601 (2015) – **UCSB**



Possible simpler solution is **three-wave mixing (3WM)**

$$\omega_p = \omega_s + \omega_i$$



No phase modulation in this case!

The inductance with non-centrosymmetric (i.e., $\chi^{(2)}$) nonlinearity is needed!

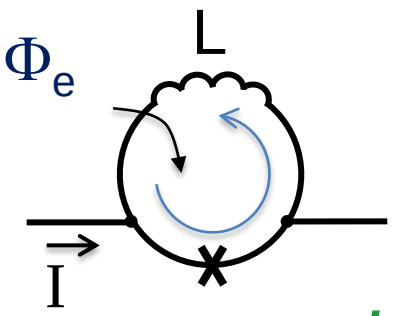
$$I = L_0^{-1}\left(\Phi - \beta\,\Phi^2 - \gamma\Phi^3 \ldots\right)$$

- Can be engineered

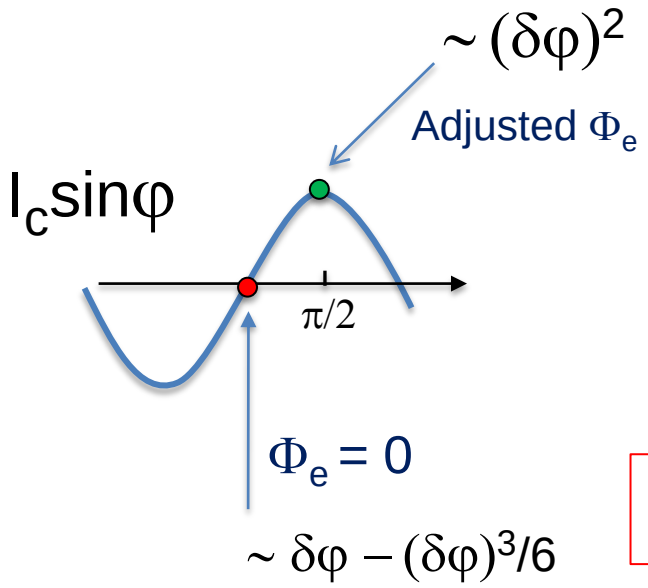
Possible solution [a.z. PRAppl. **6**, 034006 (2016)]:

One-junction SQUID

DC flux offset Φ_e



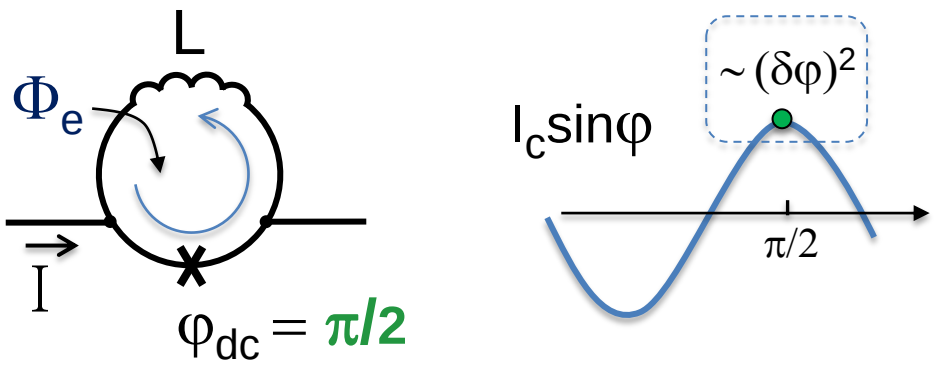
$\varphi_{dc} = \pi/2$



$\chi^{(2)} \neq 0$

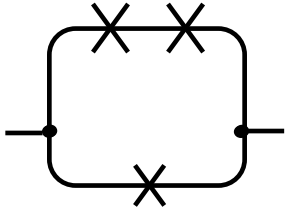
$\chi^{(3)} \neq 0$

Magnetically-controlled nonlinearity



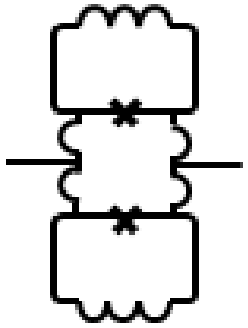
Possible modifications of the nonlinear element

Yale group [Frattini et al. APL **110**, 222603 (2017)]



- using Josephson kinetic inductance

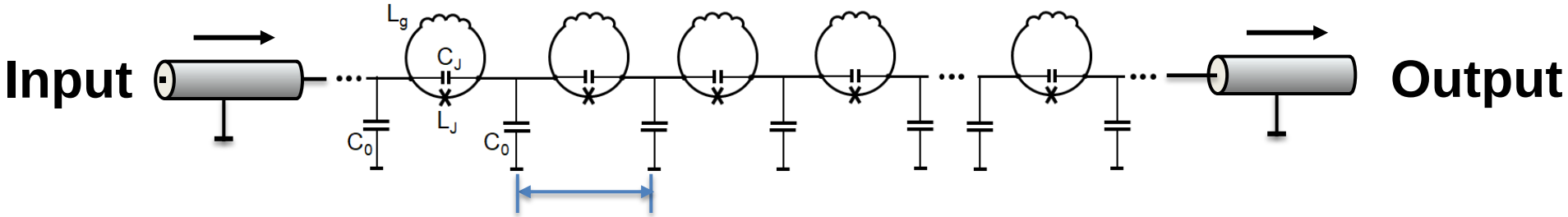
SeeQC-HYPRES group [Miano and Mukhanov, arXiv1811.02703]



- using symmetric circuit

Possible parameters of TWJPA with 3WM

N ~ 1000÷2000 – number of sections



a - size of one section ~ **20÷100 μm**

Only **one-way gain**

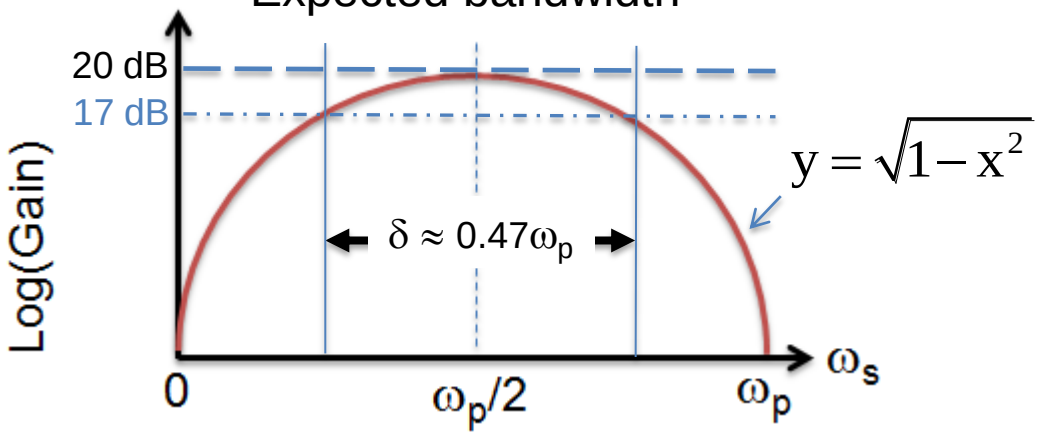
Pump frequency: **10÷15 GHz**

Wavelength: $\lambda_p \approx$ **(20÷30) a**

Total length of array **~ 50** λ_p

Velocity of wave propagation **v ~ 0.03÷0.05 c**.

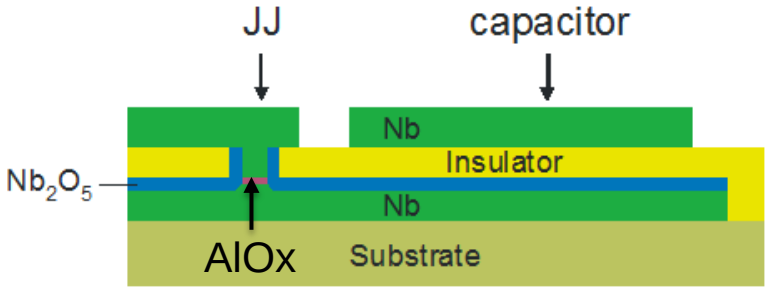
Expected bandwidth



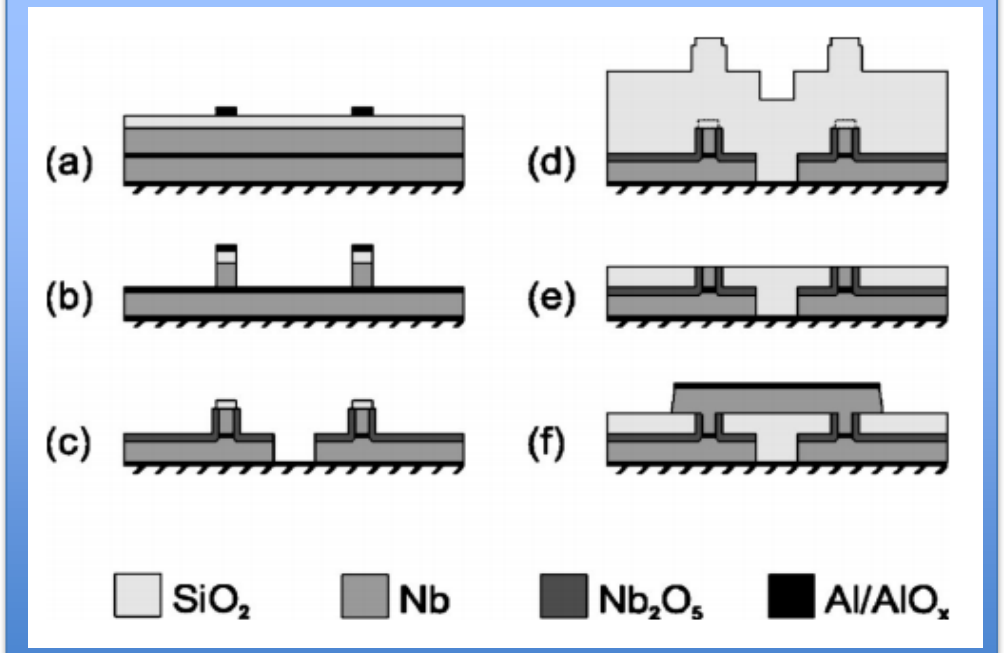
[see, e.g., P. K. Tien, JAP **29**, 1347 (1958)]

Experiment at PTB-Braunschweig (arXiv:1705.02859)

Multilayer Nb-technology



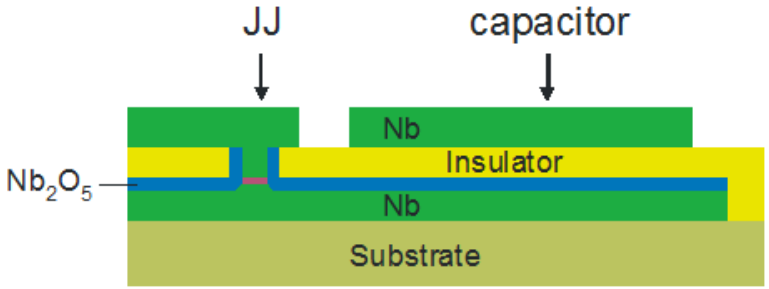
- E-beam lithography
- Deposition (Sputtering / PECVD)
- Etching (RIE / IBE)
- Chemical Mechanical Polishing (CMP)



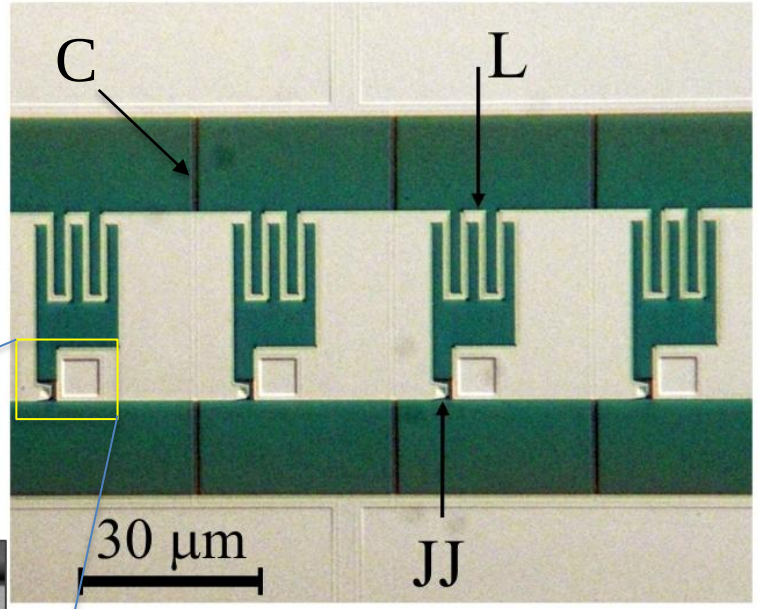
PTB process:
R. Dolata et al. J. Appl. Phys. 97, 054501 (2005)

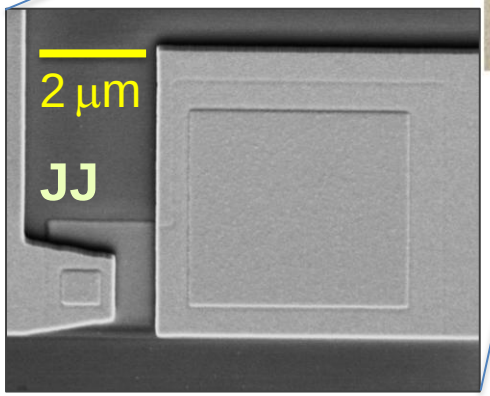
Experiment at PTB-Braunschweig (arXiv:1705.02859)

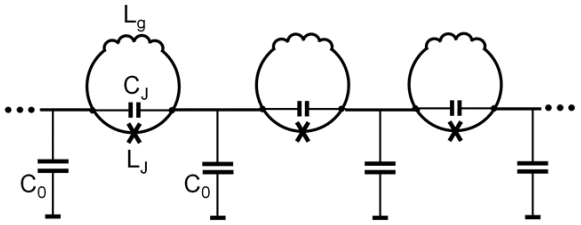
Multilayer Nb-technology



- E-beam lithography
- Deposition (Sputtering / PECVD)
- Etching (RIE / IBE)
- Chemical Mechanical Polishing (CMP)







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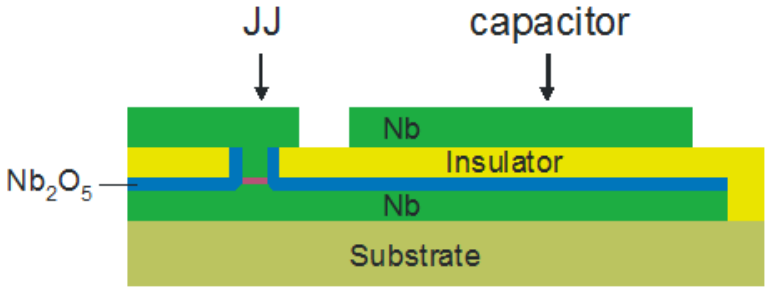
Multilayer Nb-technology



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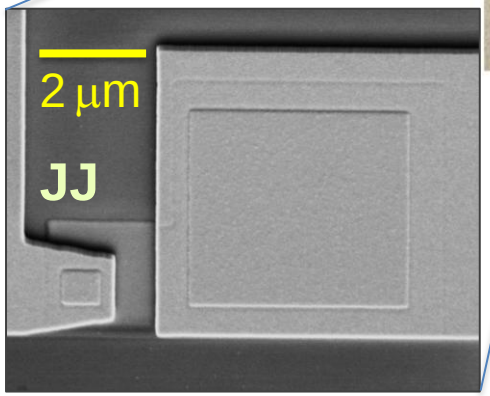
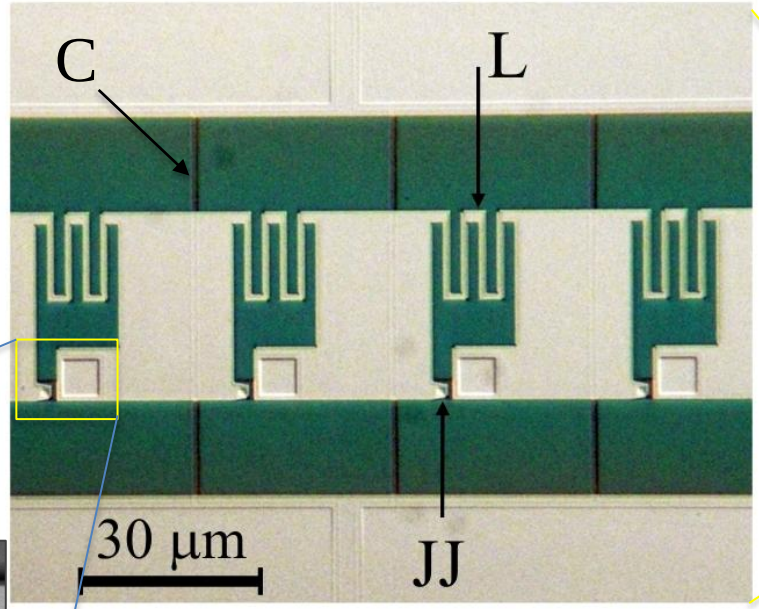
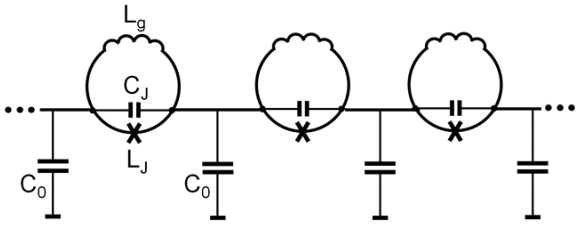
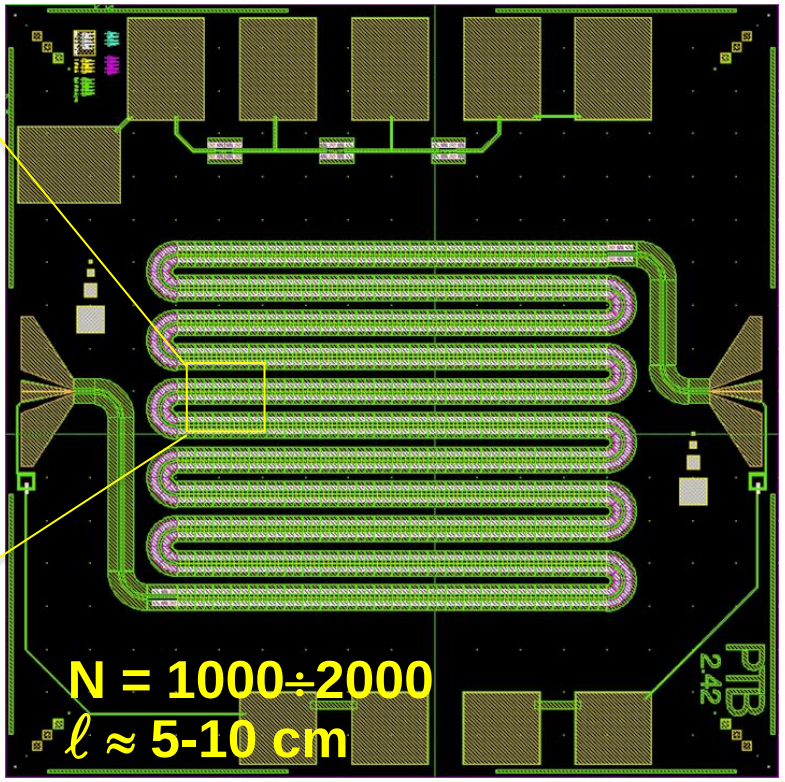




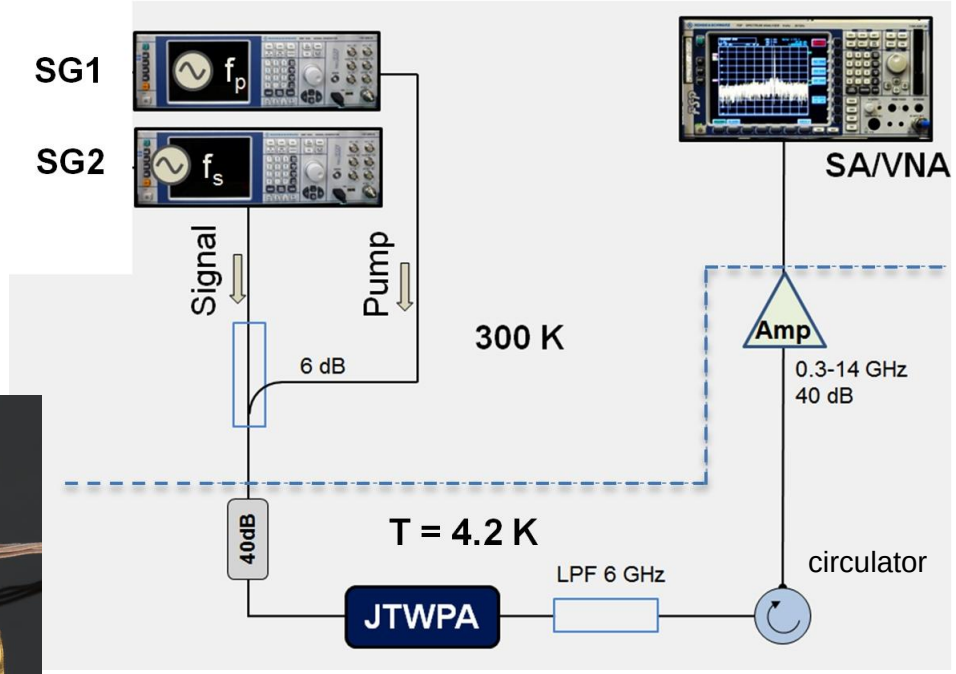


1 cm × 1 cm Si/SiO_x chip

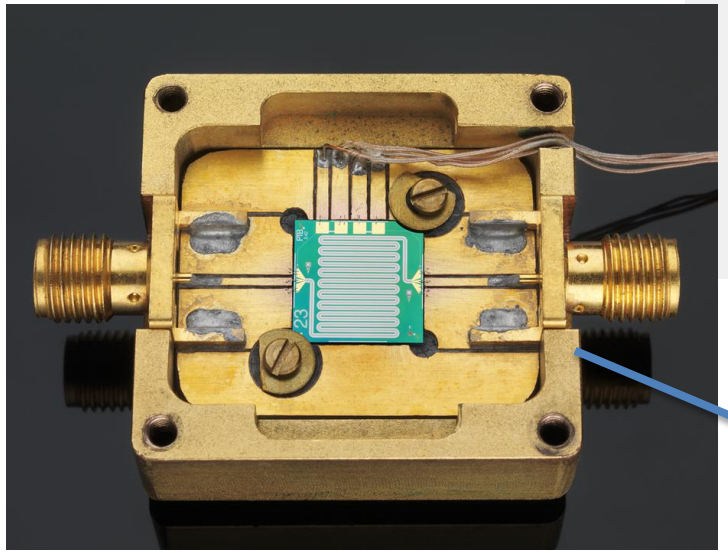


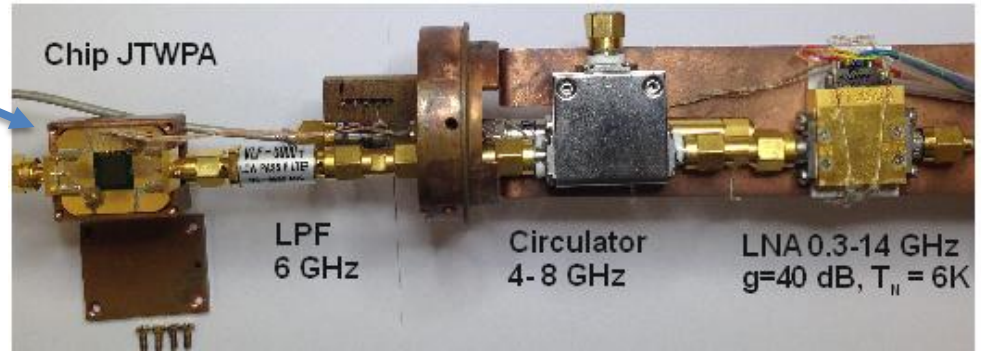
Measurements @ T = 4.2 K

Setup



Transmission measurements



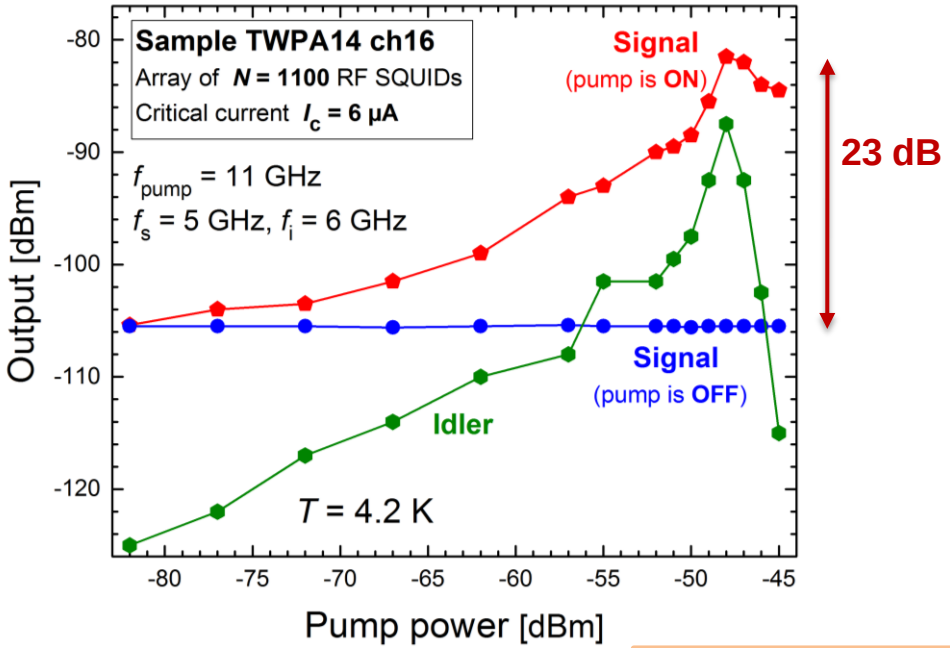


Cryogenic part
(simple)

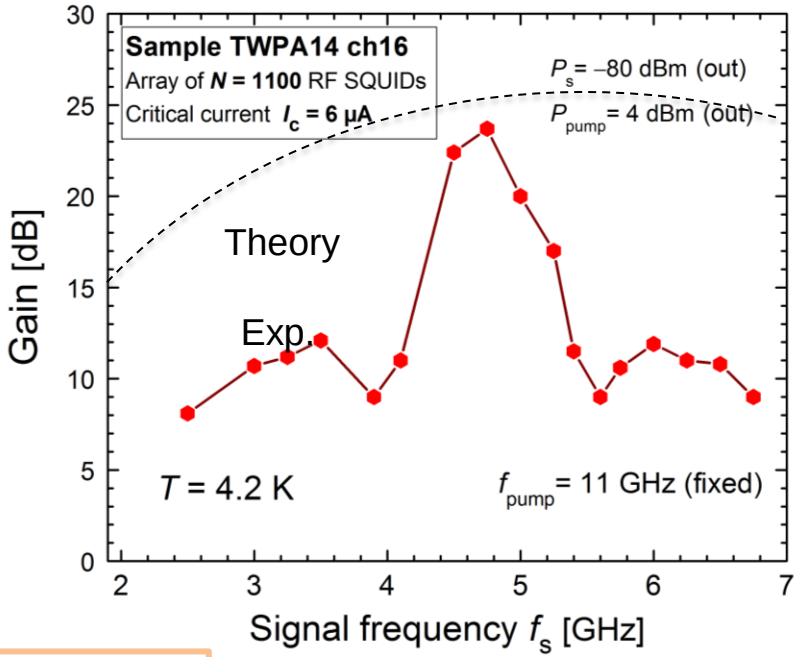
Measurements @ PTB, T = 4.2 K

(arXiv:1705.02859)

Gain



Bandwidth



Bandwidth has to be improved!

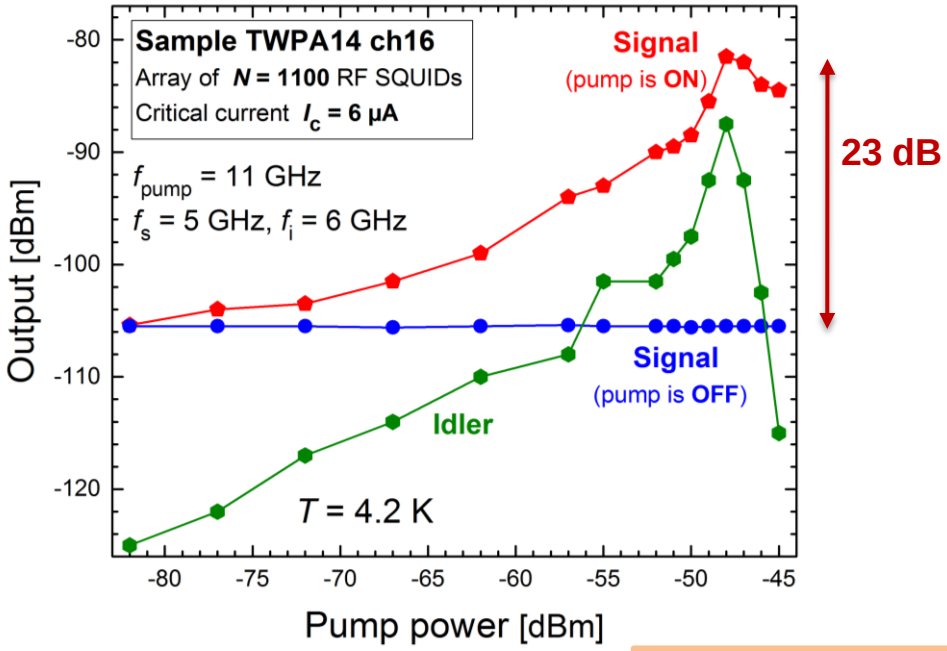
Noise temperature should be identified!

Measurements at mK – in progress

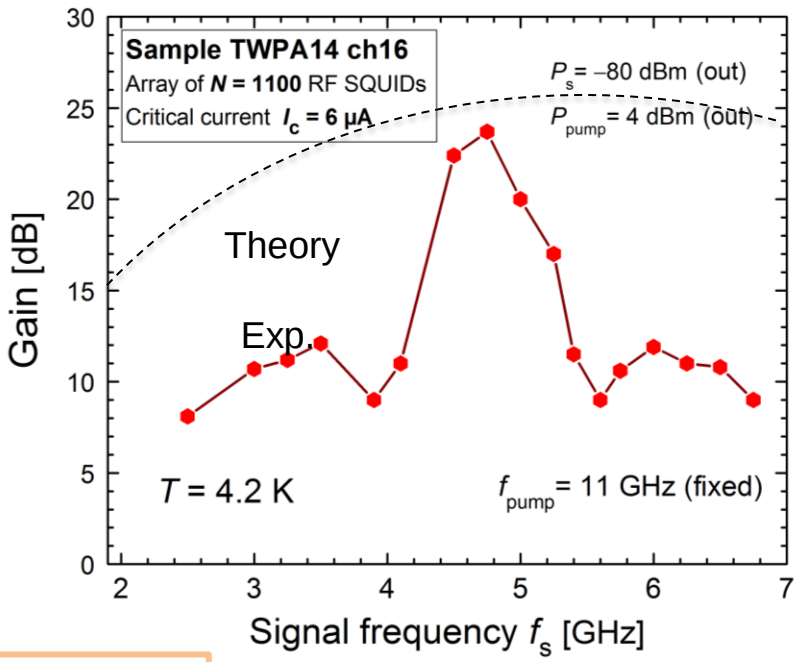
Measurements @ PTB, T = 4.2 K

(arXiv:1705.02859)

Gain



Bandwidth



Bandwidth has to be improved!

Noise temperature should be identified!

Measurements at mK – in progress

+ Encouraging recent results by **SeeQC-Hypres** (reported at ASC, Oct. 2018)

12-17 dB, 4 GHz arXiv 1811.02703

In quantum case…

Production of microwave biphotons out of quantum vacuum

“black body“

50 Ω

$$k_B T \ll \frac{\hbar\omega_p}{2}$$

$$\hbar\omega_p = \hbar\omega_1 + \hbar\omega_2$$

Entangled pairs of photons
(two-mode squeezing)

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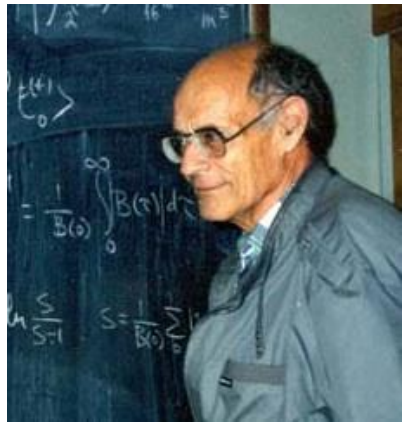
Spontaneous parametric down-conversion (SPDC)

In optics

D. N. Klyshko (1967) – prediction



$\chi^{(2)}$ - crystals:
LiNbO3, LiTaO3, BBO etc.

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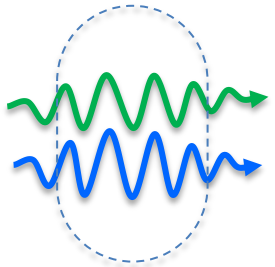
T W J P A



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Cryptographic

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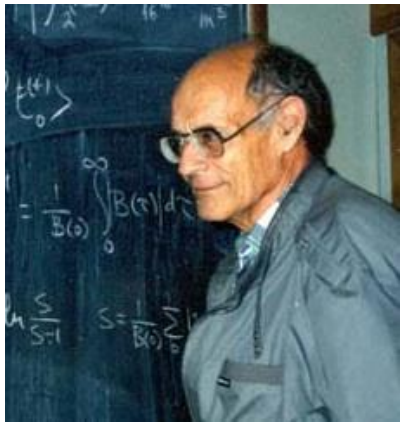


$\chi^{(2)}$ - crystals:
LiNbO3, LiTaO3, BBO etc.

- Quantum metrology
- Quantum computing
- Quantum information science
- Cryptographic applications

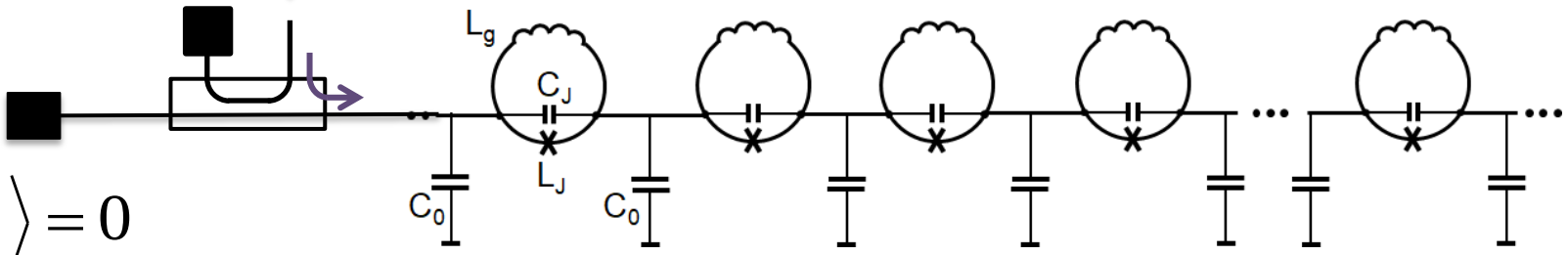
TWJPA - quantum regime of operation

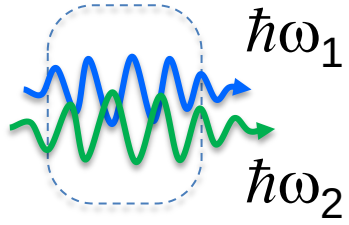
Filtered pump

ω_p

Input:

$\langle a_{in}^{\dagger} a_{in} \rangle = 0$





collinear photons

Output field:

$$a_{out}(\omega_1) = \sqrt{G}\, e^{ik_1\ell} a_{in}(\omega_1) + \sqrt{G-1}\, e^{ik_1\ell} a_{in}^{\dagger}(\omega_2)$$

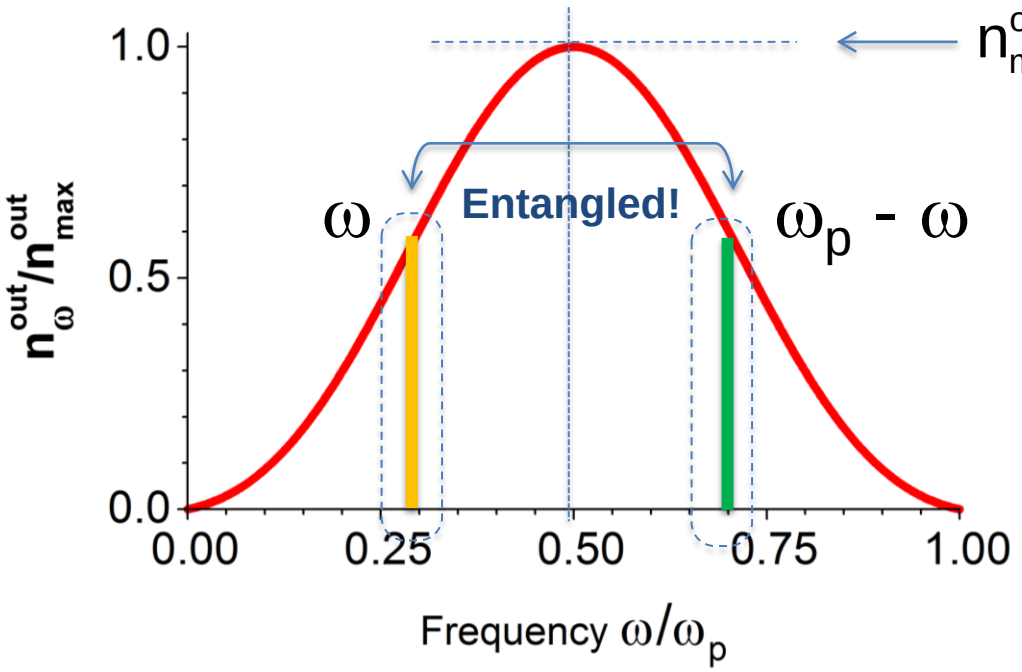
gain

$\omega_1 + \omega_2 = \omega_p$
$k_1 + k_2 = k_p$

Broadband emission is due to **broadband phase-matching**!

Shape of output spectrum

Example for gain G = 20 dB



$$n_{max}^{out} = \sqrt{(G-1)G} \approx 10^2$$

Evidence of entanglement?...

Possible prove of two-photon correlation

(1) Cauchy-Schwarz inequality for two-mode intensity correlators, b = a_{out}

$$\left[g^{(2)}_{\omega,\omega'}\right]^2 \leq g^{(2)}_{\omega}\, g^{(2)}_{\omega'} \qquad (*)$$

where $$g^{(2)}_{\omega,\omega'} = \frac{\langle \hat{b}^\dagger_\omega \hat{b}_\omega \hat{b}^\dagger_{\omega'} \hat{b}_{\omega'} \rangle}{\langle \hat{b}^\dagger_\omega \hat{b}_\omega \rangle \langle \hat{b}^\dagger_{\omega'} \hat{b}_{\omega'} \rangle}, \quad g^{(2)}_{\omega} = \frac{\langle \hat{b}^\dagger_\omega \hat{b}^\dagger_\omega \hat{b}_\omega \hat{b}_\omega \rangle}{\langle \hat{b}^\dagger_\omega \hat{b}_\omega \rangle^2}.$$

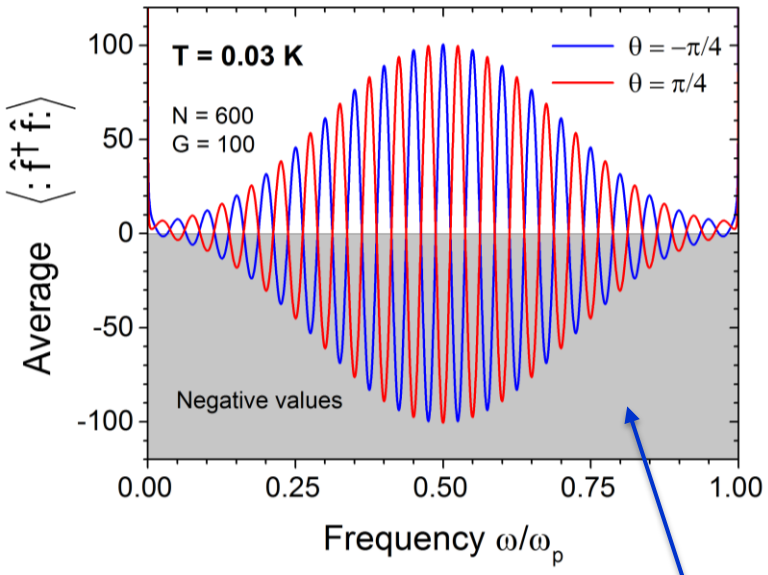
Eq. $(*)$ is violated, because (at T = 0):

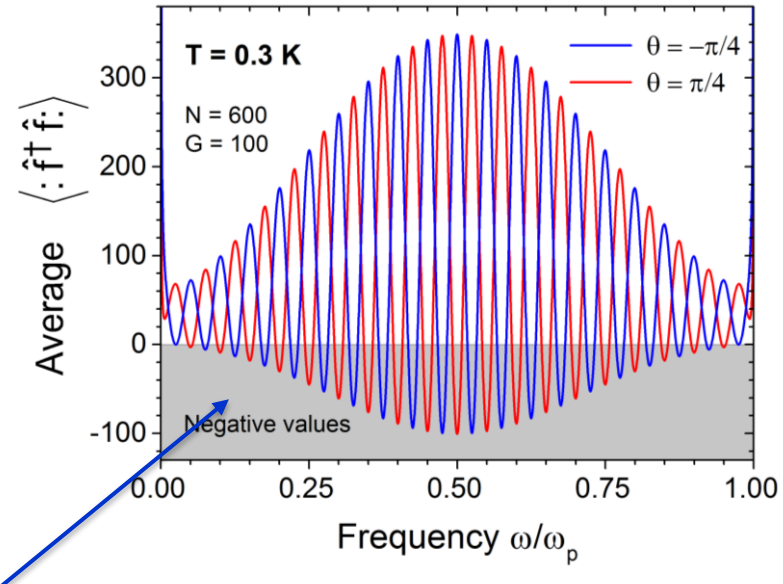
$$g^{(2)}_{\omega,\omega'} = 2 + \frac{1}{\sinh^2 gN} = 2 + \frac{1}{\langle \hat{b}^\dagger_\omega \hat{b}_\omega \rangle} \quad \text{and} \quad g^{(2)}_{\omega} = g^{(2)}_{\omega'} = 2.$$

Possible prove of two-photon correlation

Ordered average of herm. operators $\langle : \hat{f}^\dagger \hat{f} : \rangle \geq 0$ $\leftarrow$ always in classical case

Choice: two-mode squeezing $\hat{f}_\theta = \frac{1}{2}\left(e^{i\theta}\hat{b}_\omega + e^{-i\theta}\hat{b}^\dagger_\omega\right) + \frac{i}{2}\left(e^{i\theta}\hat{b}_{\omega'} - e^{-i\theta}\hat{b}^\dagger_{\omega'}\right)$



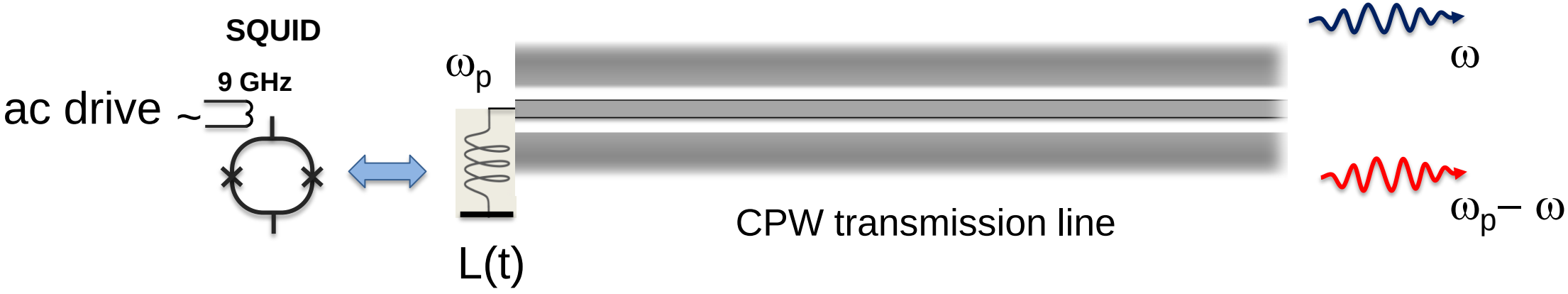


Can be measured applying homodyne detection!

Violation of inequality $\langle : \hat{f}^\dagger \hat{f} : \rangle \geq 0$, i.e. **non-classical light!**

… compare with **Dynamical Casimir Effect (DCE)**

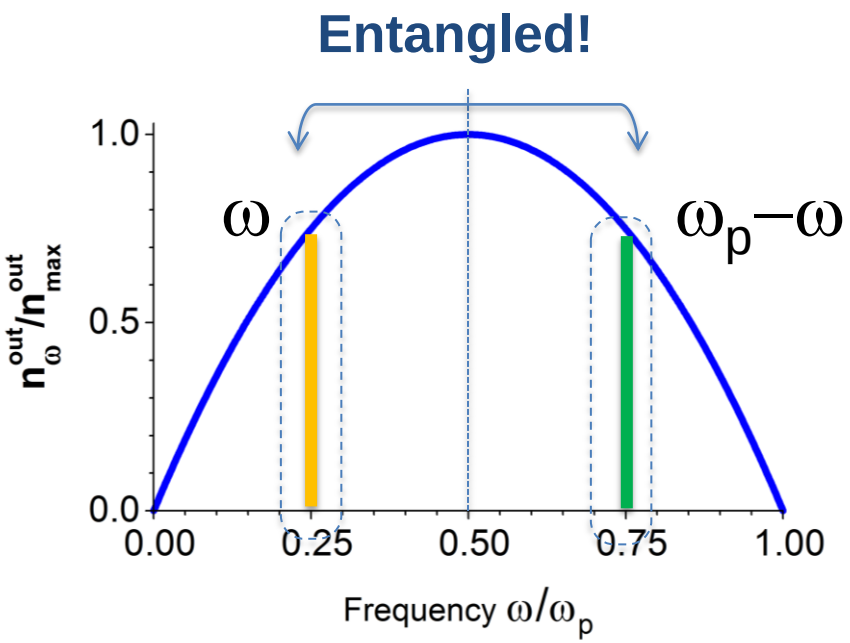
Experiment at CTH-Göteborg [Nature (2011); arXiv:1802.05529]



Time-dependent **boundary** = **moving mirror** in optics

Production of microwave biphotons (DCE)

Shape of output spectrum



$$n^{out} / n^{out}_{max} = \frac{4\omega(\omega_p - \omega)}{\omega_p^2}$$

Intensity

$$n^{out}_{max} \approx 3.5 \times 10^{-3}$$

More than 4 orders weaker than SPDC in TWJPA!

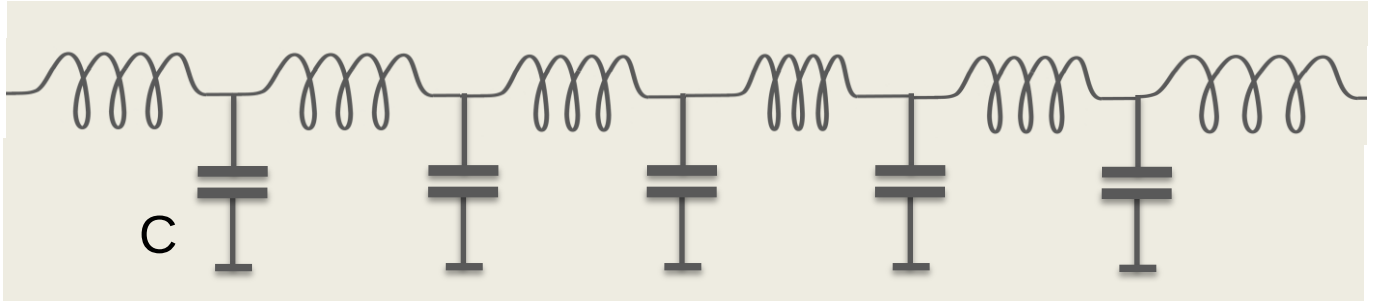
New concept of TWPA with 3WM

[arXiv:1804.09109]

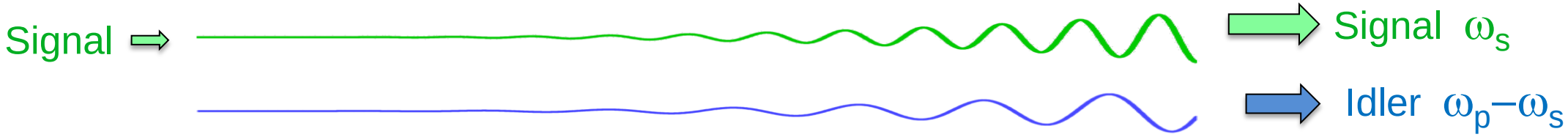
A wave-like variation of the distributed inductance:

$$L^{-1}(x,t) = \left[1 + m\,\sin(k_p x - \omega_p t)\right] L_0^{-1}$$

Produced by external wave!



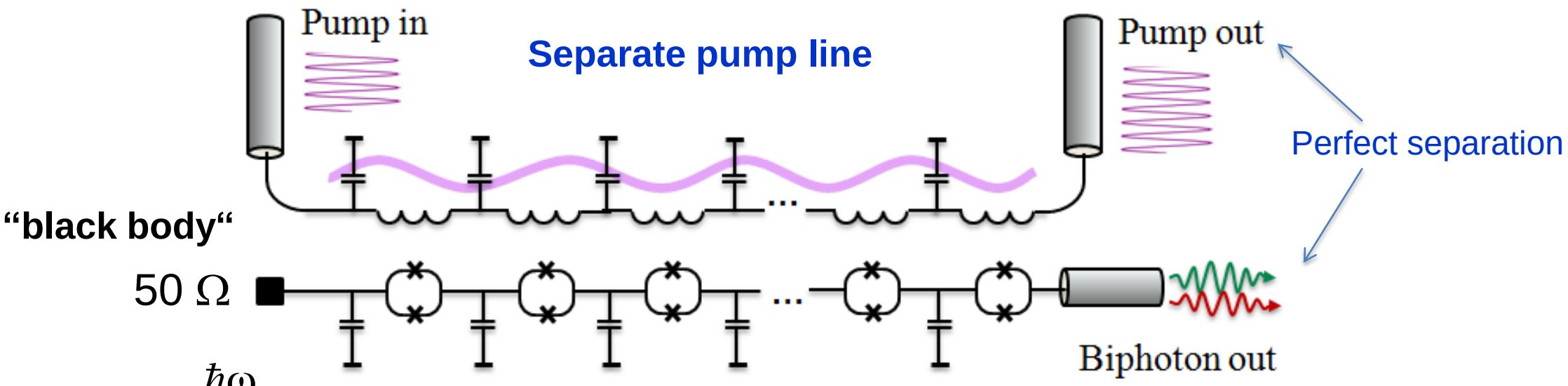
Linear transmission line!



…and good phase matching: $\mathbf{k_s + k_i = k_s}$!

Another variant of the microwave biphoton source

[arXiv:1804.09109]



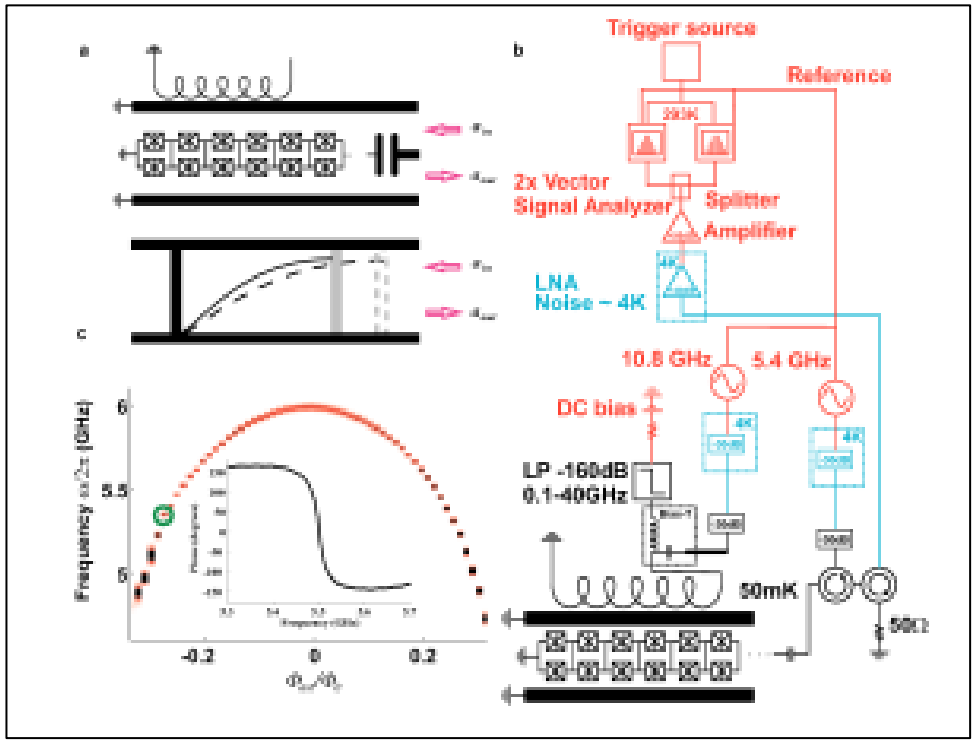
$$k_B T \ll \frac{\hbar\omega_p}{2}$$

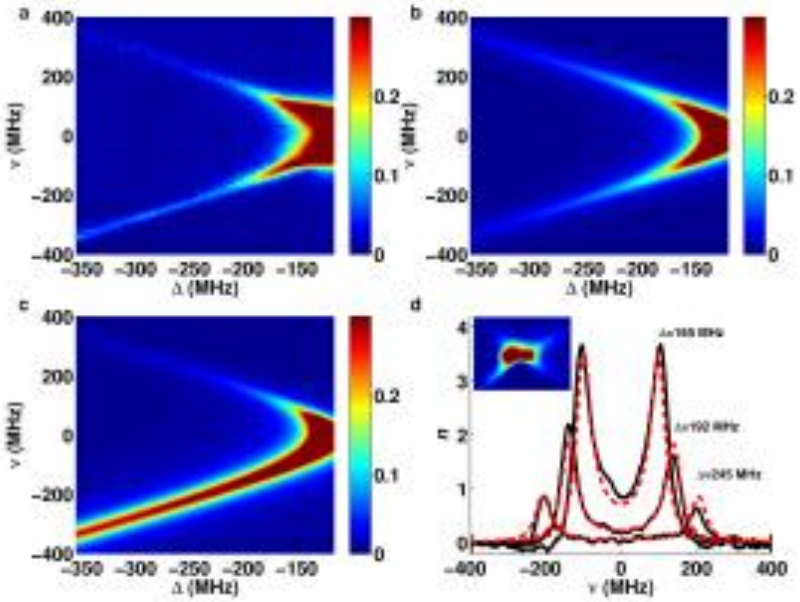
Array of SQUIDs operation in linear regime

Principle of operation: modulation of the line refraction index in a traveling-wave fashion

Dynamical Casimir Effect (DCE) in superconducting circuits

P. Lähteenmäki et al. PNAS **110**, 4234 (2012) – Aalto Helsinki





Principle of operation: periodic modulation of the refraction index in cavity

Conclusion and outlook

1. Remarkable $\chi^{(2)}$ Josephson (meta)material available
2. Proof-of-concept experiment at T = 4.2 K (promising!)

To be done next:

- Quantum-limited performance, squeezing
- Integration with SQUID, SET, qubit, etc.
- Two-mode broadband entanglement

Special thanks to:

Thomas Weimann (PTB)

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Tom Dixon (RHUL)

Connor Shelly (RHUL)

Emanuele Enrico (INRiM)

Luca Fasolo (INRiM)

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EMPIR EURAMET

The EMPIR initiative is co-funded by the European Union's Horizon 2020 research and innovation programme and the EMPIR Participating States

Extra slides

In optics

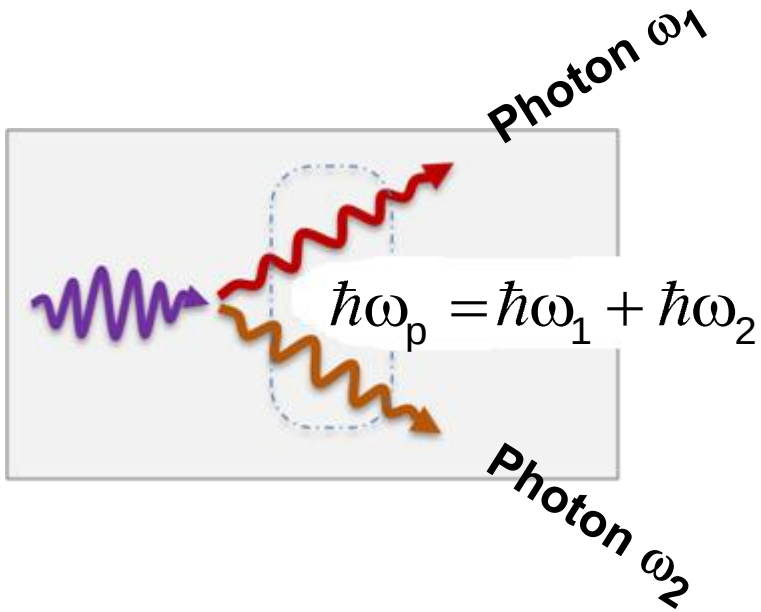
Spontaneous parametric down-conversion (SPDC):

generation of entangled photons out of vacuum [discovered in 1960s]

SPDC in $\chi^{(2)}$- crystals: LiNbO3, LiTaO3, BBO etc. **... but $\chi^{(2)}$- fibers not available!**

$\rightarrow$ **cavity configuration**

Photon of pump



$$\hbar\omega_p = \hbar\omega_1 + \hbar\omega_2$$

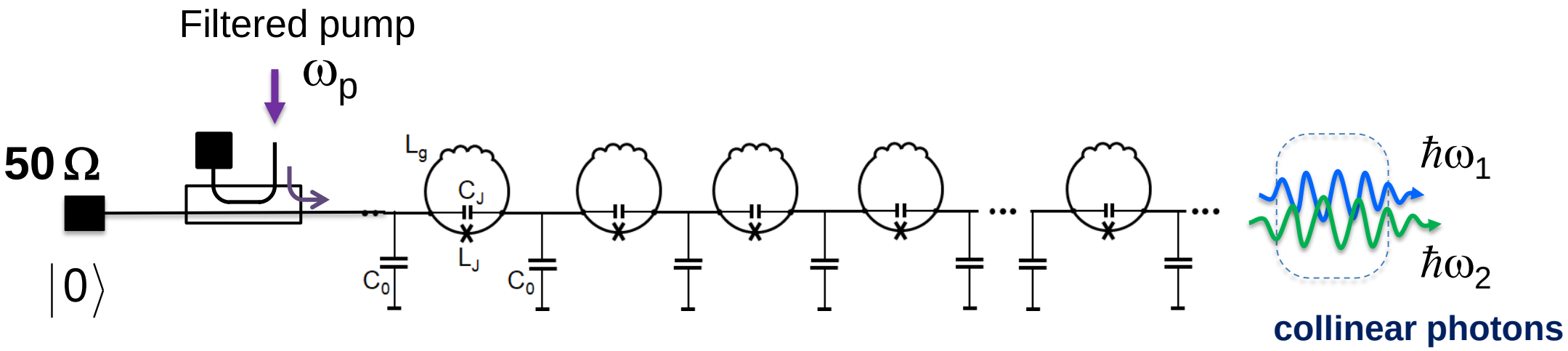
$$\hbar\vec{k}_p = \hbar\vec{k}_1 + \hbar\vec{k}_2$$

Phase matching condition

$\rightarrow$ **not collinear photons!**

Possible experiment with JTWPA…

TWJPA - quantum regime of operation



Output field:

$$b_{\omega 1} = \sqrt{G}\, e^{ik_1\ell} a_{in}(\omega_1) + \sqrt{G-1}\, e^{ik_1\ell} a_{in}^{\dagger}(\omega_2),$$

$$b_{\omega 2} = \sqrt{G}\, e^{ik_2\ell} a_{in}(\omega_2) + \sqrt{G-1}\, e^{ik_2\ell} a_{in}^{\dagger}(\omega_1)$$

gain

$\omega_1 + \omega_2 = \omega_p$
$k_1 + k_2 = k_p$

Broadband emission is due to **broadband phase-matching**!