Quantum Synchronization

L.C. Kwek

In collaborations with Sameer Sonar, Michal Hajdusek, Manas Mukherjee, Rosario Fazio, Vlatko Vedral, and Sai Vinjanampathy, Hermanni Heimonen, Andy Chia and Daryl Mok

Moscow State University, September , 5, 2019

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- Introduction: What is synchronization? And perhaps what is not?
- Quantum synchronization: Some recent results on quantum synchronization…
- Squeezing effects: Our recent contribution….
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DISCUSSION AND CORRESPONDENCE

A CASE OF SYNCHRONIC BEHAVIOR IN PHALANGIDÆ

A RECENT article in this journal by Wallace Craig on "Synchronism in the Rhythmic Activities of Animals" recalls to mind an observation that I made near Austin, Texas, in 1909. At the time of the observation I made some field notes from which the following description is taken.

While engaged in hunting various species of rock lizards I located a vast colony of "harvestmen," which I identified as belonging to the genus *Liobunum*, resting during the day on the under side of an overhanging shelf of rock on a precipitous hillside. In a somewhat circular area of nearly five feet in diameter the harvestmen were packed closely together in almost unbelievable numbers. I estimated that there were between one and two thousand in the colony. When I first saw them they were all hanging from the ceiling, as it were, perfectly motionless, but when I came within about six feet of them they began a curious rhythmic dance. Without changing their foot-holds they raised their bodies up and down at the rate of about three times a second, and, curiously enough, the movement of the entire lot was in the most perfect unison. This performance was kept up for over a minute and then stopped gradually as though from exhaustion. I then poked a few of the nearest individuals with a stick and these immediately resumed the rhythmic up-and-down movement, which spread quickly over the whole group, but died down in less than half a minute. When I once more stirred up a

beginning, but became so after a few seconds.

Possibly synchronic flashing in fire-flies may be explained as the result of a somewhat similar transmission of stimuli. One flash stimulates others, which at first might lag slightly; but soon a synchronism is built up in a limited region, such as one bush or one tree. Such a synchronism might be transmitted to a whole field.

It would be interesting to know whether any other naturalist has observed the type of behavior herewith described for the Phalangidæ.

H. H. NEWMAN

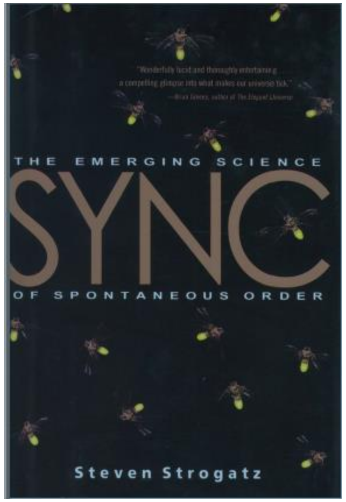
UNIVERSITY OF CHICAGO

THE SUPPOSED SYNCHRONAL FLASHING OF FIREFLIES

I WAS very much interested in reading the article by H. A. Allard, entitled "The Synchronal Flashing of Fireflies," which appeared in SCIENCE, November 17, 1916. Some twenty years ago I saw, or thought I saw, a synchronal or simultaneous flashing of fireflies (Lampyridæ). I could hardly believe my eyes, for such a thing to occur among insects is certainly contrary to all natural laws. However, I soon solved the enigma. The apparent phenomenon was caused by the twitching or sudden lowering and raising of my eyelids. The insects had nothing whatsoever to do with it. Many times in the past twenty years I have proved that my solution was correct.

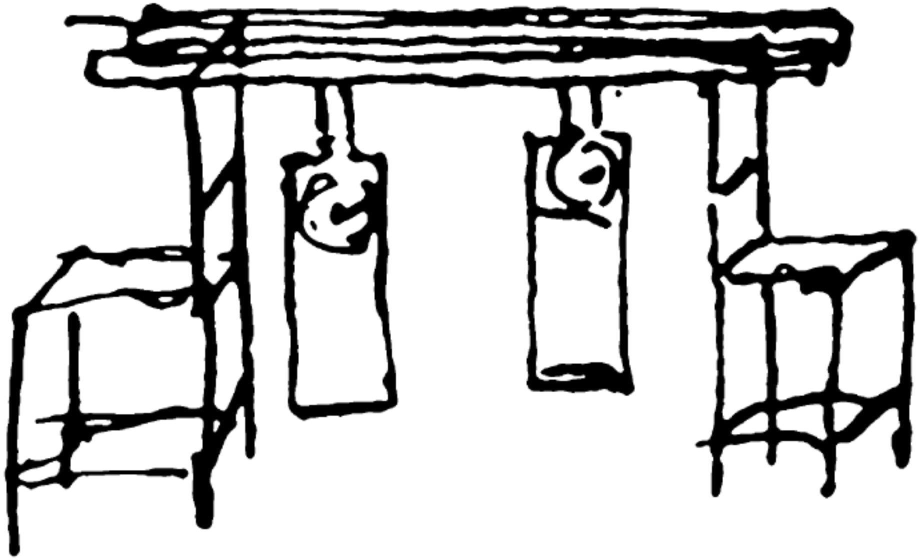
PHILIP LAURENT

TRIMMED MAGAZINES AND EFFICIENCY EXPERTS



The naturalist, Hugh Smith, who had lived in Thailand from 1923 to 1934 and witnessed the displays countless times, wrote in exasperation that "some of the published explanations are more remarkable than the phenomenon itself." But he confessed that he too was unable to offer any explanation…Strogatz, Sync, p.

Christiaan Huygens (1629–1695)



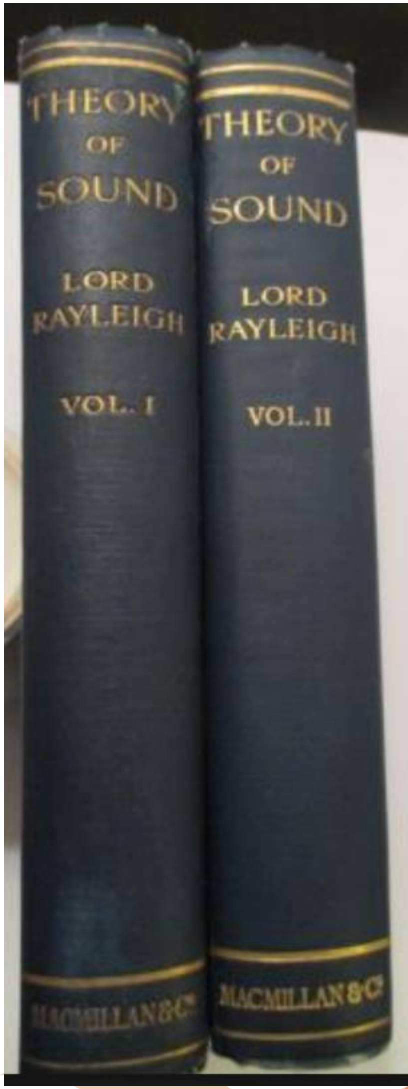
Original drawing of Christiaan Huygens illustrating his experiments with two pendulum clocks placed on a common support.

A1.1 A letter from Christiaan Huygens to his father, Constantyn Huygens[1]

26 February 1665.

While I was forced to stay in bed for a few days and made observations on my two clocks of the new workshop, I noticed a wonderful effect that nobody could have thought of before. The two clocks, while hanging [on the wall] side by side with a distance of one or two feet between, kept in pace relative to each other with a precision so high that the two pendulums always swung together, and never varied. While I admired this for some time, I finally found that this happened due to a sort of sympathy: when I made the pendulums swing at differing paces, I found that half an hour later, they always returned to synchronism and kept it constantly afterwards, as long as I let them go. Then, I put them further away from one another, hanging one on one side of the room and the other one fifteen feet away. I saw that after one day, there was a difference of five seconds between them and, consequently, their earlier agreement was only due to some sympathy that, in my opinion, cannot be caused by anything other than the imperceptible stirring of the air due to the motion of the pendulums. Yet the clocks are inside closed boxes that weigh, including all the lead, a little less than a hundred pounds each. And the vibrations of the pendulums, when

[1] Translation from French by Carsten Henkel.

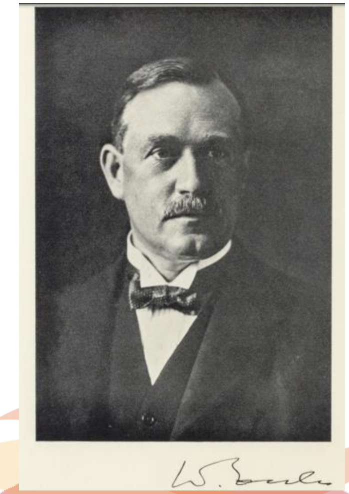


Sir John William Strutt, Lord Rayleigh (1842–1919).

Sir Edward Victor Appleton (1892–1965).

Lord Rayleigh described the interesting phenomenon of synchronization in acoustical systems as follows:

When two organ-pipes of the same pitch stand side by side, complications ensue which not unfrequently give trouble in practice. In extreme cases the pipes may almost reduce one another to silence. Even when the mutual influence is more moderate, it may still go so far as to cause the pipes to speak in absolute unison, in spite of inevitable small differences.



On 17 February 1920 W. H. Eccles and J. H. Vincent applied for a British Patent confirming their discovery of the synchronization property of a triode generator – a rather simple electrical device based on a vacuum tube that produces a periodically alternating electrical current…. Eccles and Vincent coupled two generators which had slightly different frequencies and demonstrated that the coupling forced the systems to vibrate with a common frequency

The synchronization phenomenon was used to stabilize the frequency of a powerful generator with the help of one which was weak but very precise.

Balthasar van der Pol (1889–1959)

$$\frac{d^2x}{dt^2} - \mu(1 - x^2)\frac{dx}{dt} + x = 0,$$

E.M.F., $E_0 \sin \omega t$, is applied to it, currents and potential differences occur in the system the frequencies of which are whole submultiples of the frequency of the applied E.M.F., *e.g.* $\omega/2$, $\omega/3$, $\omega/4$ up to $\omega/40$.

To this end one can make use of the remarkable synchronising properties of relaxation-oscillations,

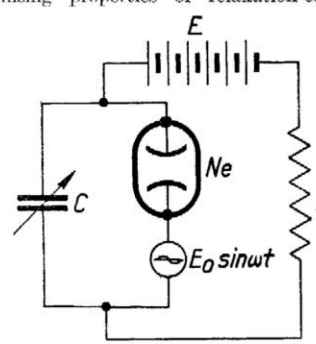


FIG. 1.

i.e. oscillations the time period of which is determined by the approximate expression $T = \pi/2\,C\,R$, a relaxation time (Balth. van der Pol, "On Relaxation Oscillations," *Phil. Mag.*, p. 978, 1926; also *Zeitschr. f. hochfreq. Technik*, **29**, 114; 1927).

Let *Ne* in Fig. 1 represent a neon glow lamp, *R* a resistance of the order of a few megohms, *C* a variable condenser of ap-

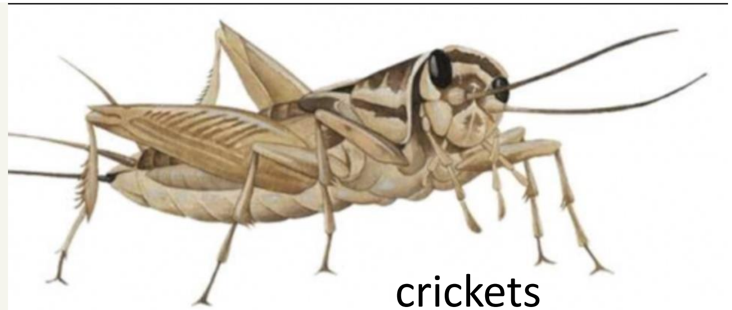
up to 1000/40 sec.$^{-1}$. In some recent experiments it was found possible to obtain a frequency demultiplication up to the ratio 1 : 1/200. Often an irregular noise is heard in the telephone receivers before the frequency jumps to the next lower value. However, this is a subsidiary phenomenon, the main effect being the regular frequency demultiplication. It may be noted that while the production of harmonics, as with frequency multiplication, furnishes us with tones determining the musical major scale, the phenomenon of frequency-division renders the musical minor scale audible. In fact, with a properly chosen 'fundamental' ω, the turning of the condenser in the region of the third to the sixth subharmonic strongly reminds one of the tunes of a bagpipe.

In conclusion, we give in Fig. 2 the measured time periods (which are thus found to be a series of discrete subharmonics) as a function of the setting of the condenser *C*. The dotted line in the figure gives the frequency with which the system oscillates in the absence of the applied alternating E.M.F. The shaded parts correspond to those settings of the condenser where an irregular noise is heard. In the actual experiment the resistance *R* was, for ease of adjustment, replaced by a diode. The experiment, however, succeeds just as well with an ohmic resistance *R*. Obviously the same experiment succeeds with all systems capable of producing relaxation-oscillations such as described in the papers quoted.

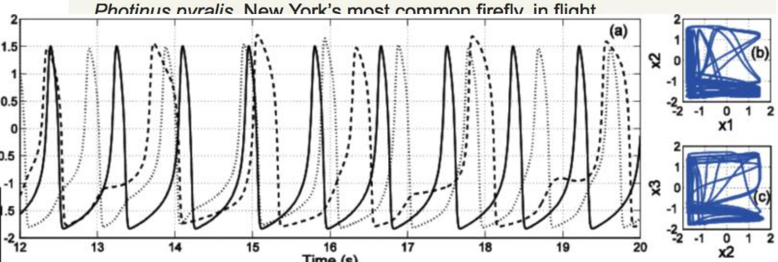
BALTH. VAN DER POL.
J. VAN DER MARK.
Natuurkundig Laboratorium der
N. V. Philips' Gloeilampenfabrieken,
Eindhoven, Aug. 5.

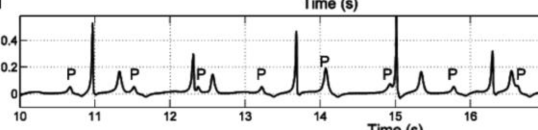
Many classical synchronization phenomena in Nature

Photinus pyralis, New York's most common firefly, in flight



crickets

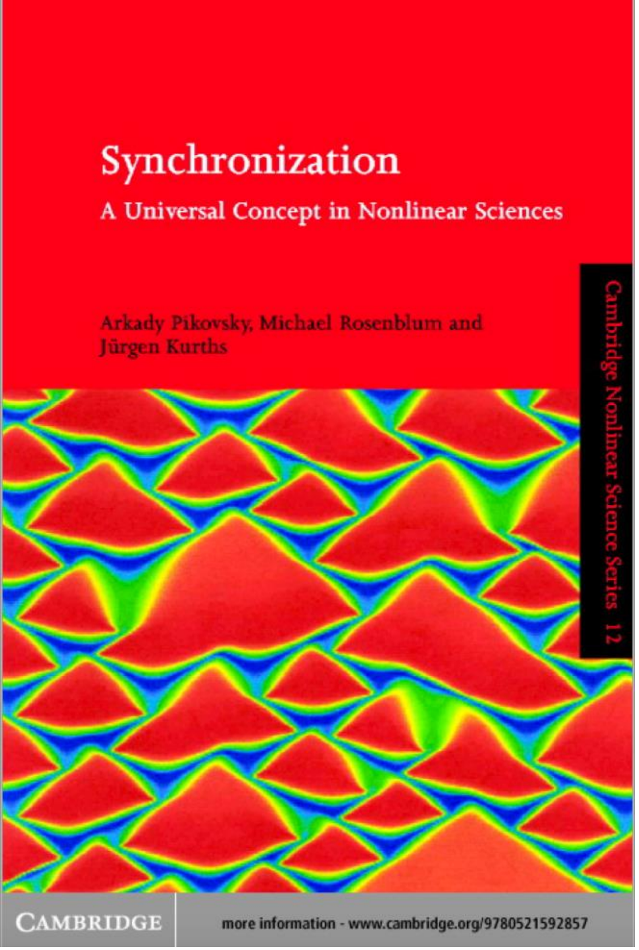




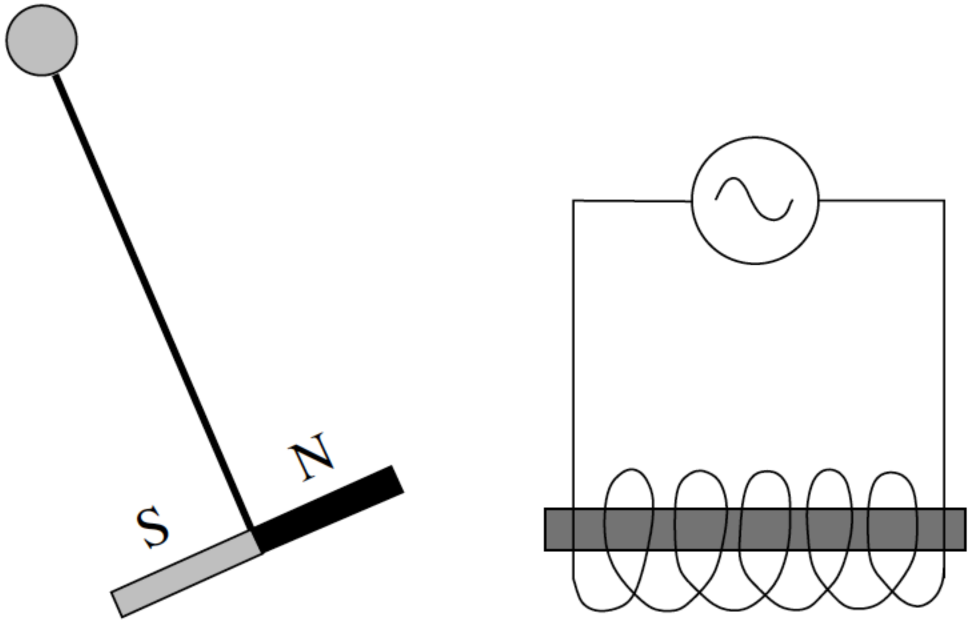
Neurons and pacemakers

Milleniums bridge

Strogatz, et. al, Nature, **438**, 43-44 (2005)



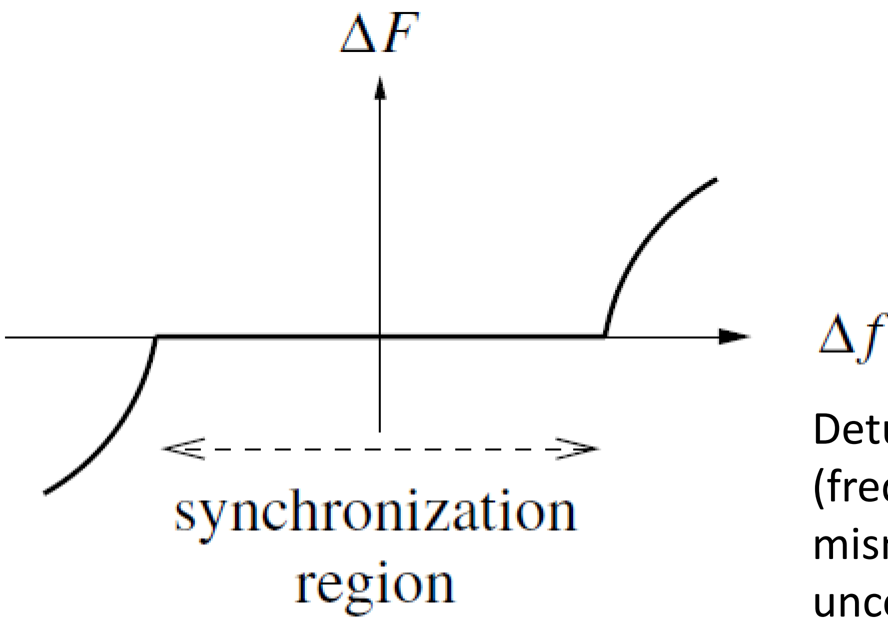
Not synchronization!



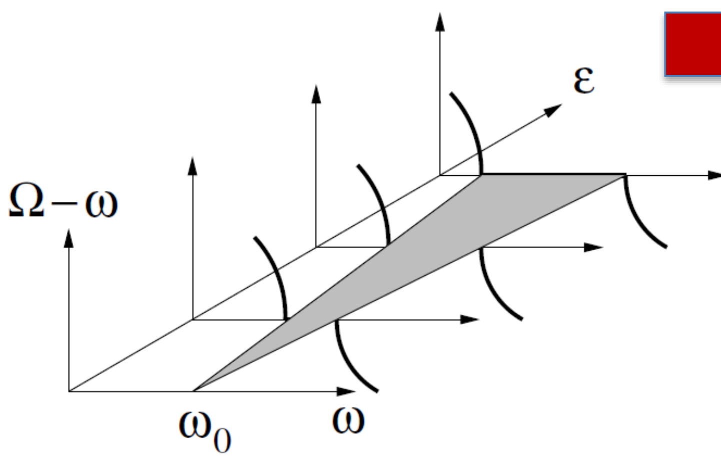
The pendulum is not a self-sustained system and cannot oscillate continuously: being kicked, it starts to oscillate, but this free oscillation decays due to friction forces.

Synchronization is different from resonance

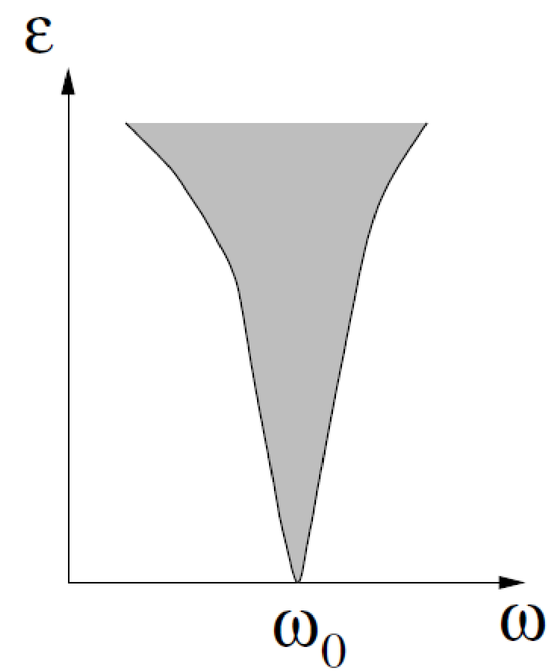
Difference in frequency of two coupled oscillators



Detuning (frequency mismatch) of uncoupled systems



Arnold Tongue



PHYSICAL REVIEW A **97**, 043406 (2018)

Synchronization of a self-sustained cold-atom oscillator

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Nonlinear oscillations and synchronization phenomena are ubiquitous in nature. We study the synchronization of self-oscillating magneto-optically trapped cold atoms to a weak external driving. The oscillations arise from a dynamical instability due the competition between the screened magneto-optical trapping force and the interatomic repulsion due to multiple scattering of light. A weak modulation of the trapping force allows the oscillations of the cloud to synchronize to the driving. The synchronization frequency range increases with the forcing amplitude. The corresponding Arnold tongue is experimentally measured and compared to theoretical predictions. Phase locking between the oscillator and drive is also observed.

DOI: 10.1103/PhysRevA.97.043406

PHYSICAL

Synchronization o

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[5]Université Côte d'Azur, CNI

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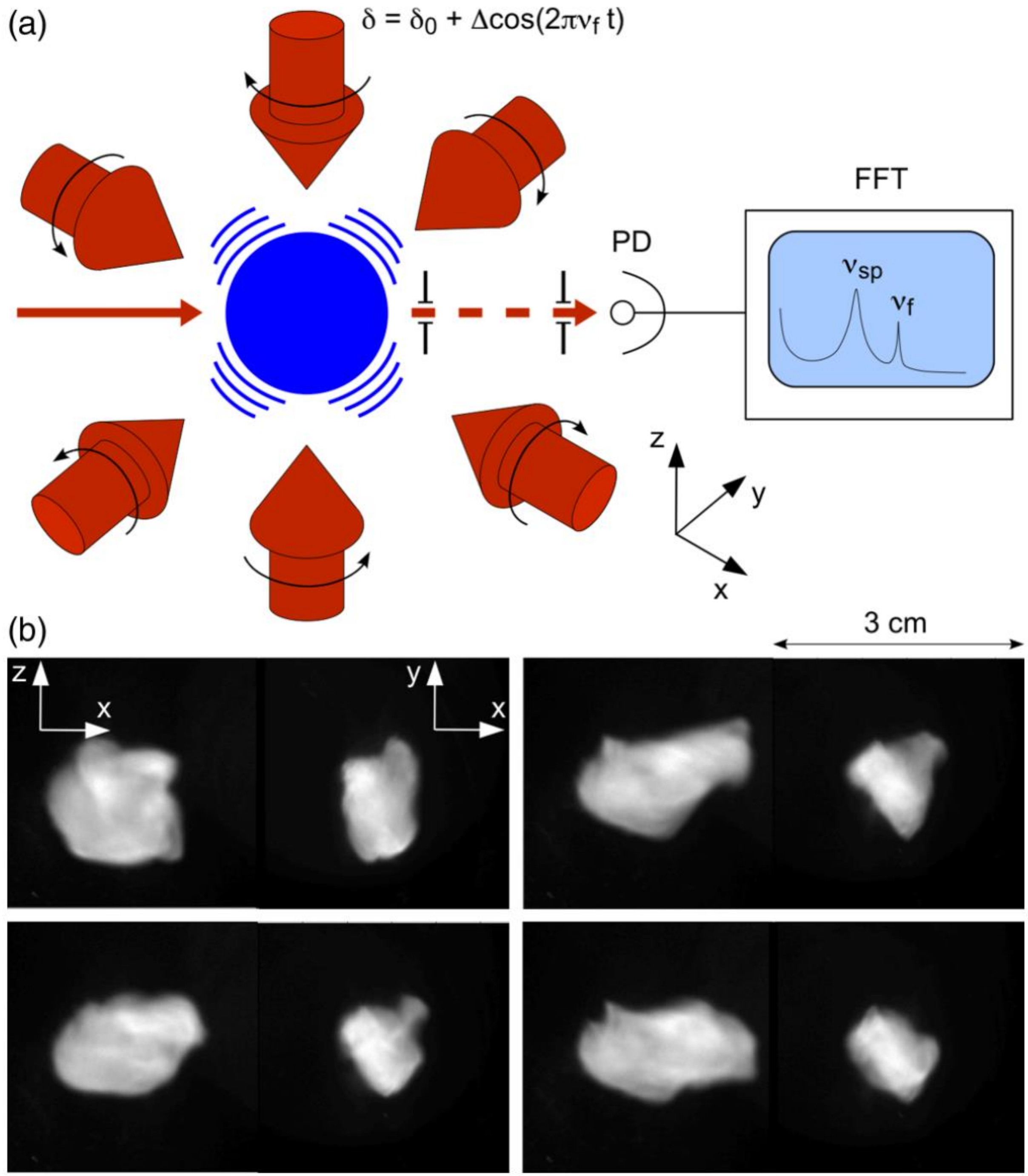

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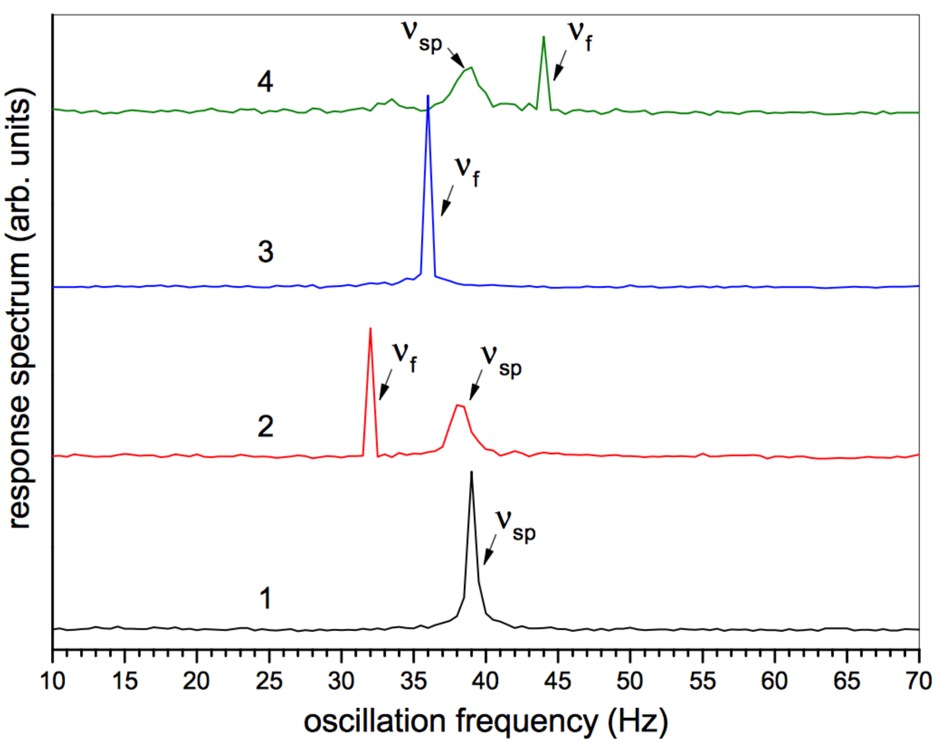

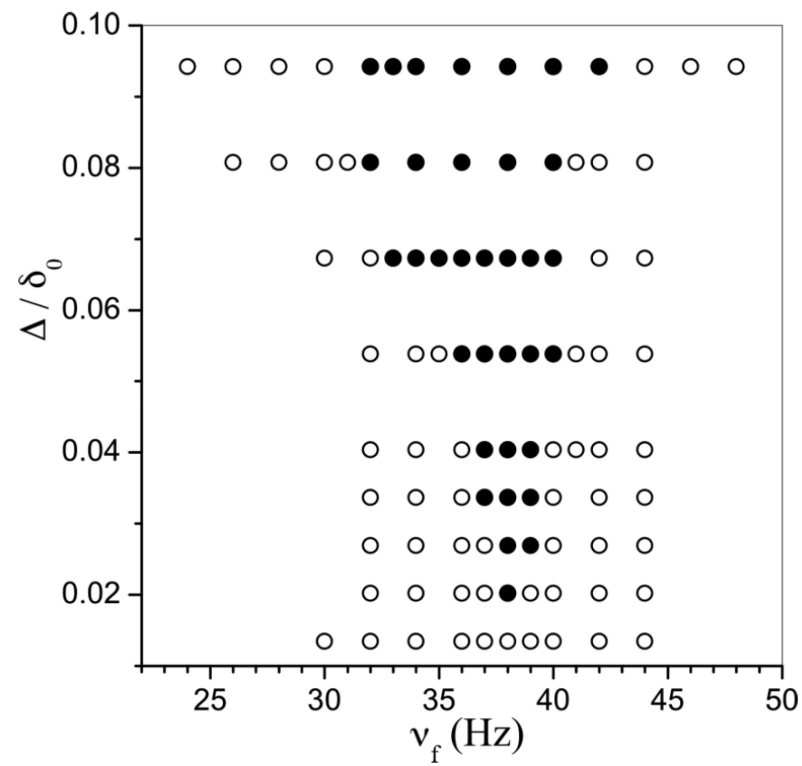

PRL **112,** 094102 (2014) PHYSICAL REVIEW LETTERS week ending 7 MARCH 2014

Quantum Synchronization of a Driven Self-Sustained Oscillator

Stefan Walter, Andreas Nunnenkamp, and Christoph Bruder
Department of Physics, University of Basel, Klingelbergstrasse 82, CH-4056 Basel, Switzerland
(Received 26 July 2013; revised manuscript received 16 October 2013; published 6 March 2014)

Synchronization is a universal phenomenon that is important both in fundamental studies and in technical applications. Here we investigate synchronization in the simplest quantum-mechanical scenario possible, i.e., a quantum-mechanical self-sustained oscillator coupled to an external harmonic drive. Using the power spectrum we analyze synchronization in terms of frequency entrainment and frequency locking in close analogy to the classical case. We show that there is a steplike crossover to a synchronized state as a function of the driving strength. In contrast to the classical case, there is a finite threshold value in driving. Quantum noise reduces the synchronized region and leads to a deviation from strict frequency locking.

DOI: 10.1103/PhysRevLett.112.094102 PACS numbers: 05.45.Xt, 42.65.-k, 07.10.Cm

$$\frac{d\rho}{dt} = -i[-\Delta\hat{b}^\dagger\hat{b} + i\Omega(\hat{b} - \hat{b}^\dagger), \rho] + \gamma_1\mathcal{D}[\hat{b}^\dagger]\rho + \gamma_2\mathcal{D}[\hat{b}^2]\rho,$$

Classical Van der Pol Equation

$$\frac{d^2x}{dt^2} - \mu(1-x^2)\frac{dx}{dt} + x = 0$$

?

$$\gamma_1 >> \gamma_2$$

$$\frac{d\rho}{dt} = -i[\Delta\hat{b}^\dagger\hat{b} + iF(\hat{b}-\hat{b}^\dagger), \rho] + \gamma_1 D[\hat{b}^\dagger]\rho + \gamma_2 D[\hat{b}^2]\rho$$

Quantum Van der Pol Equation?

Strictly speaking: this Equation is a quantum version of the Stuart Landau Equation….behavior of nonlinear oscillator near Hopf bifurcation

$$\mathcal{D}[\hat{\mathcal{O}}]\rho = \hat{\mathcal{O}}\rho\hat{\mathcal{O}}^\dagger - \frac{1}{2}\{\hat{\mathcal{O}}^\dagger\hat{\mathcal{O}}, \rho\}$$

For microscopic derivation:
ArXiv: 1711.07376

Quantum Synchronization

PRL **112,** 094102 (2014) PHYSICAL REVIEW LETTERS week ending 7 MARCH 2014

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Back to Bruder's paper

Synchron… in technical ap… ario possible, i.e… ing the power s… ing in close analogy to the classical case. We show that there is a steplike crossover to a synchronized state as a function of the driving strength. In contrast to the classical case, there is a finite threshold value in driving. Quantum noise reduces the synchronized region and leads to a deviation from strict frequency locking.

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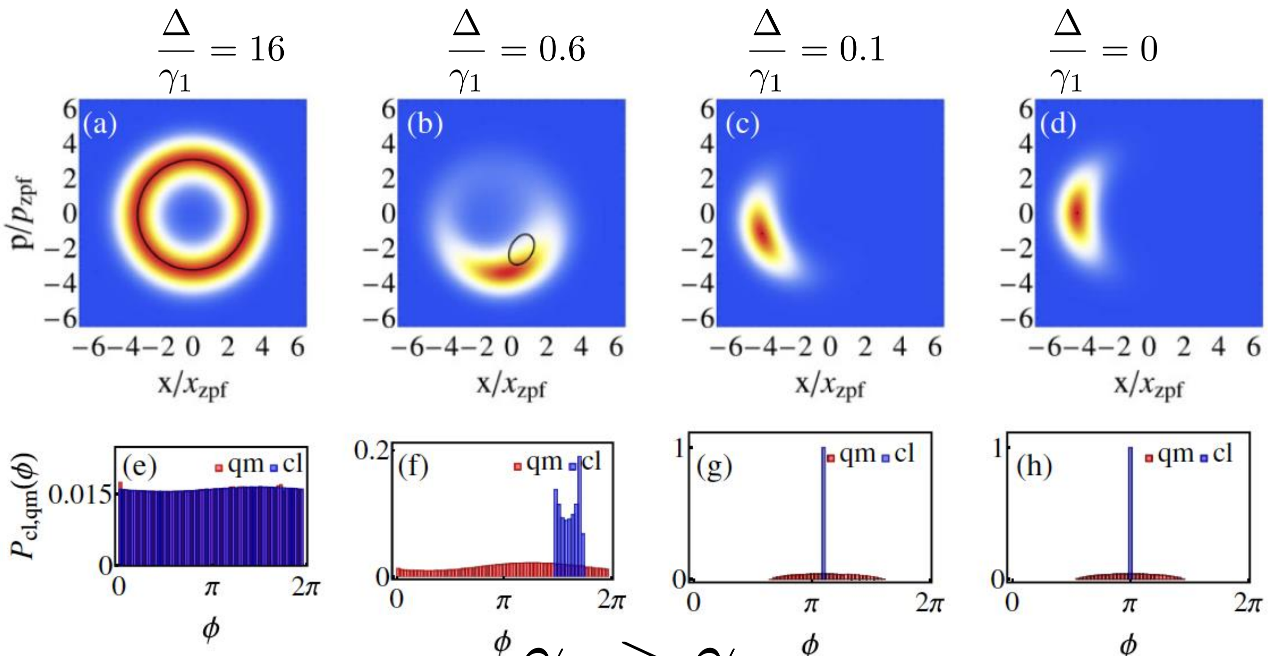
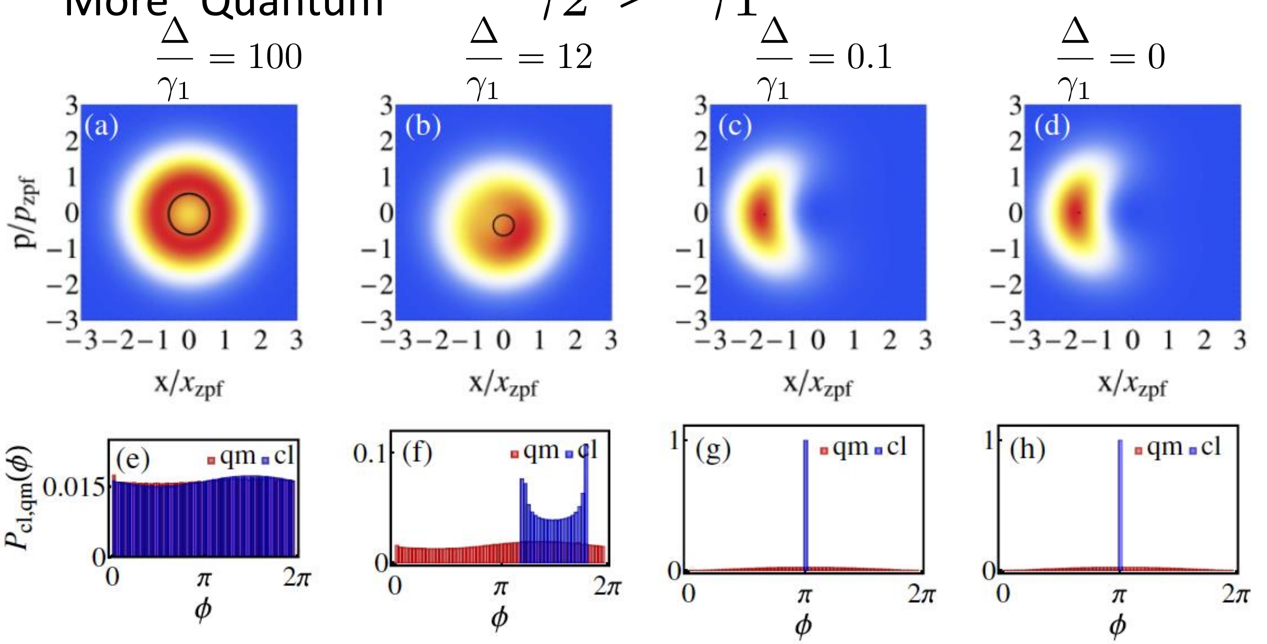
$$\frac{d\rho}{dt} = -i[-\Delta \hat{b}^\dagger \hat{b} + i\Omega(\hat{b} - \hat{b}^\dagger), \rho] + \gamma_1 \mathcal{D}[\hat{b}^\dagger]\rho + \gamma_2 \mathcal{D}[\hat{b}^2]\rho,$$

More “Classical” $\gamma_2 < \gamma_1$



More “Quantum” $\gamma_2 > \gamma_1$



PRL **111,** 234101 (2013) PHYSICAL REVIEW LETTERS week ending 6 DECEMBER 2013

Quantum Synchronization of Quantum van der Pol Oscillators with Trapped Ions

Tony E. Lee and H. R. Sadeghpour

ITAMP, Harvard-Smithsonian Cen

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The van der Pol oscillator is the p
nonlinear behavior in biological and o
affect phase locking of one or many v
robust in the quantum model than in th
suited to simulate van der Pol oscilla
motional modes. We provide realisti
technology.

DOI: 10.1103/PhysRevLett.111.234101

The van der Pol (vdP) oscillator was origi
ceived in 1920 to describe nonlinear behavior i
tube circuits [1]. Since then, it has been the
countless works, and is now a textbook model in
dynamics [2]. As the prototypical self-sustained
that can phase-lock with an external drive or
oscillators [3], the vdP oscillator has been used

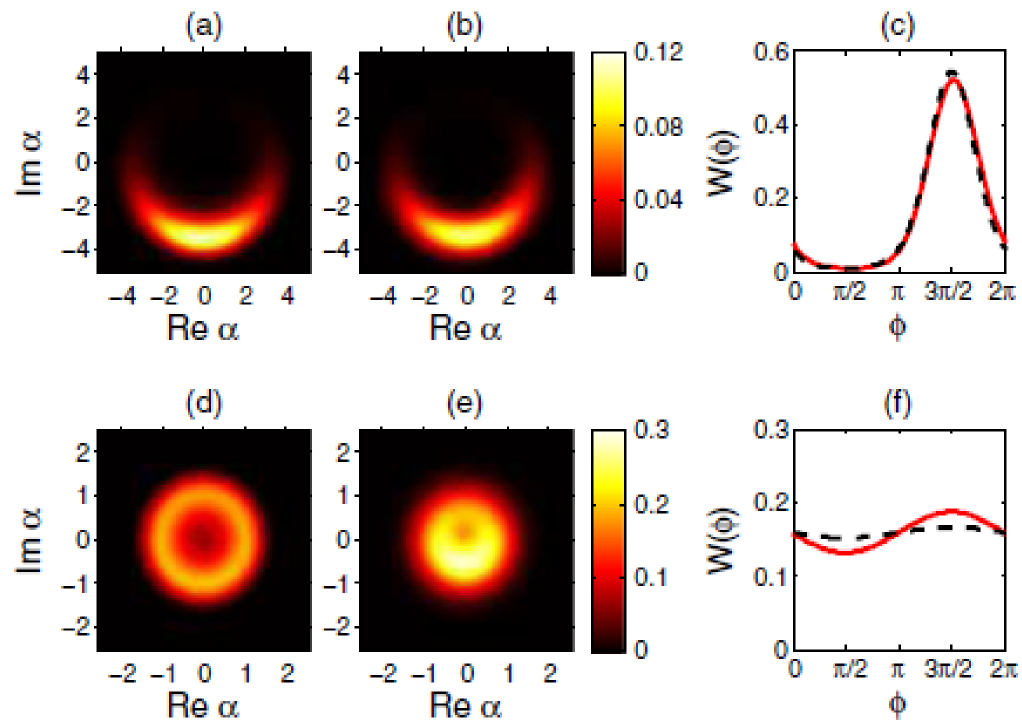


FIG. 2 (color online). Wigner function for an oscillator with external drive $E = \kappa_1$. (a)–(c) Classical limit with $\kappa_2 = 0.05\kappa_1$: (a) W_c, (b) W_q, and (c) both W_c (black, dashed line) and W_q (red, solid line) after integrating out $|\alpha|$. (d)–(f) Same, but for the quantum limit with $\kappa_2 = 20\kappa_1$.

Ann. Phys. (Berlin) 527, No. 1–2, 131–138 (2015) / DOI 10.1002/andp.201400144

annalen der physik

Original Paper

Quantum synchronization of two Van der Pol oscillators

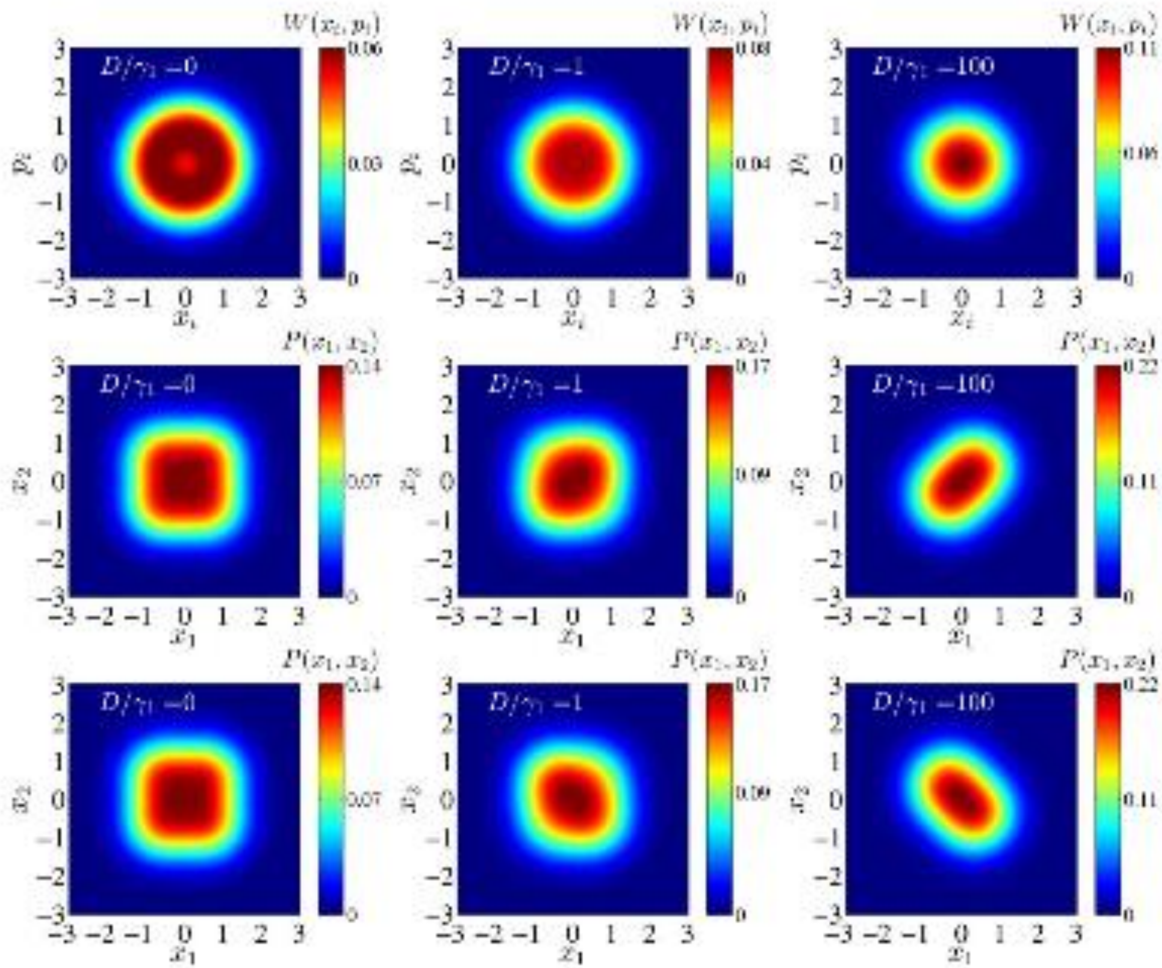
Stefan Walter, Andreas Nunnenkamp, and Christoph Bruder*

Received 27 June 2014, accepted 11
Published online 6 August 2014

Synchronization of two dissipati
cillators in the quantum regime
noise strict frequency locking is
crossover from weak to strong fr
differences to the behavior of on
cillator subject to an external dri
a possible experimental realizati
tum Van der Pol oscillators in an
described.

1 Introduction

Synchronization is a ubiquito
in a variety of physical syste
self-sustained (or limit-cycle)



Squeezing Enhances Quantum Synchronization

PHYSICAL REVIEW LETTERS **120,** 163601 (2018)

Editors' Suggestion

Squeezing Enhances Quantum Synchronization

Sameer Sonar,[1] Michal Hajdušek,[2,*] Manas Mukherjee,[2,3,4] Rosario Fazio,[5,6,2] Vlatko Vedral,[7,2]
Sai Vinjanampathy,[1,2,†] and Leong-Chuan Kwek[2,4,8,9]

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It is desirable to observe synchronization of quantum systems in the quantum regime, defined by the low number of excitations and a highly nonclassical steady state of the self-sustained oscillator. Several existing proposals of observing synchronization in the quantum regime suffer from the fact that the noise statistics overwhelm synchronization in this regime. Here, we resolve this issue by driving a self-sustained oscillator with a squeezing Hamiltonian instead of a harmonic drive and analyze this system in the classical and quantum regime. We demonstrate that strong entrainment is possible for small values of squeezing, and in this regime, the states are nonclassical. Furthermore, we show that the quality of synchronization measured by the FWHM of the power spectrum is enhanced with squeezing.

Squeezing Enhances Quantum Synchronization (2018). *Phys. Rev. Lett.* 120 163601

Quantum van der Pol equation....

$$\dot{\rho} = -i[\Delta \hat{b}^\dagger \hat{b} + iF(\hat{b} - \hat{b}^\dagger), \rho] + \gamma_1 \mathcal{D}[\hat{b}^\dagger]\rho + \gamma_2 \mathcal{D}[\hat{b}^2]\rho$$

$$H_{sq} = i\chi^{(2)}(\hat{b}^2 \hat{c}^\dagger - \hat{b}^{\dagger 2} \hat{c})$$

$$\mathcal{D}[\hat{\mathcal{O}}]\rho = \hat{\mathcal{O}}\rho\hat{\mathcal{O}}^\dagger - \frac{1}{2}\{\hat{\mathcal{O}}^\dagger \hat{\mathcal{O}}, \rho\}$$

Assume pump mode depletion is negligible, approx
$\hat{c} = \lambda \exp(-i(\omega_p t - \theta))$

ω_d is the frequency of the drive and $\Delta = \omega_0 - \omega_d$ with ω_0 as the natural frequency of the vdP

$\eta = \chi^{(2)}\lambda$ is the squeezing parameter

Set $\omega_p = 2\omega_d$

$$\hat{H}_{\text{tot}} = \Delta\hat{b}^\dagger\hat{b} + iF(\hat{b} - \hat{b}^\dagger) + i\eta(\hat{b}^2 e^{-i\theta} - \hat{b}^{\dagger 2} e^{i\theta}),$$

$$\dot{\rho} = -i[\hat{H}_{\text{tot}}, \rho] + \gamma_1 \mathcal{D}[\hat{b}^\dagger]\rho + \gamma_2 \mathcal{D}[\hat{b}^2]\rho.$$

$\eta = 0, F \neq 0$

Harmonically driven vdP

$\eta \neq 0, F = 0$

Squeezed driven vdP

For highly excited oscillator, $\gamma_1 >> \gamma_2$, replace $\langle \hat{b} \rangle = R\exp(i\phi)$

$$\dot{R} = \frac{\gamma_1}{2}R - \gamma_2 R^3 - F\cos\phi - 2\eta R\cos(2\phi - \theta),$$

$$\dot{\phi} = -\Delta + \frac{F}{R}\sin\phi + 2\eta\sin(2\phi - \theta).$$

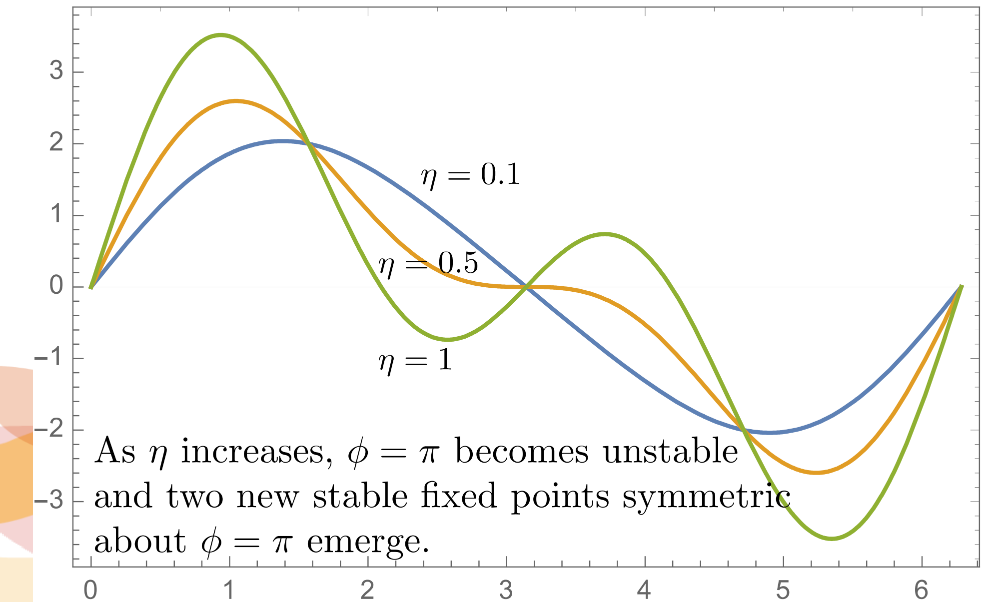
For highly excited oscillator, $\gamma_1 >> \gamma_2$, replace $\langle \hat{b} \rangle = R \exp(i\phi)$

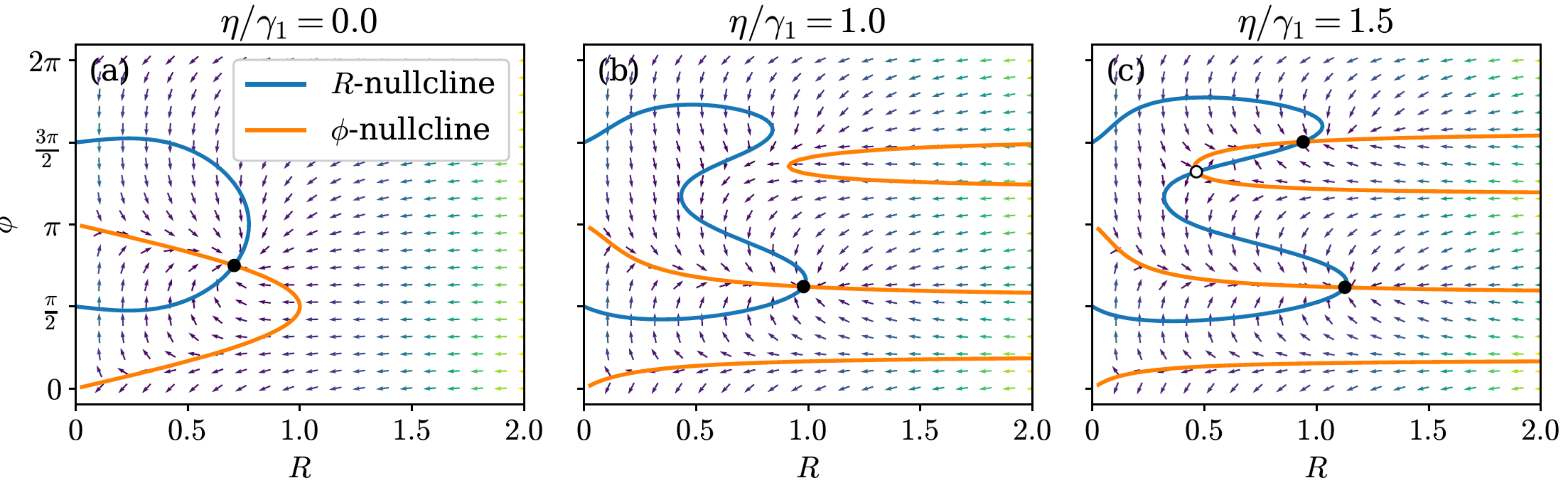
$$\dot{R} = \frac{\gamma_1}{2} R - \gamma_2 R^3 - F \cos\phi - 2\eta R \cos(2\phi - \theta),$$

$$\dot{\phi} = -\Delta + \frac{F}{R} \sin\phi + 2\eta \sin(2\phi - \theta).$$

In the simple case of driving on resonance ($\Delta = 0$) and squeezing along the position quadrature $\theta = 0$, we have pitchfork bifurcation,

$$\sin\phi_{ss}\left(\frac{F}{R_{ss}} + 4\eta\cos\phi_{ss}\right) = 0$$

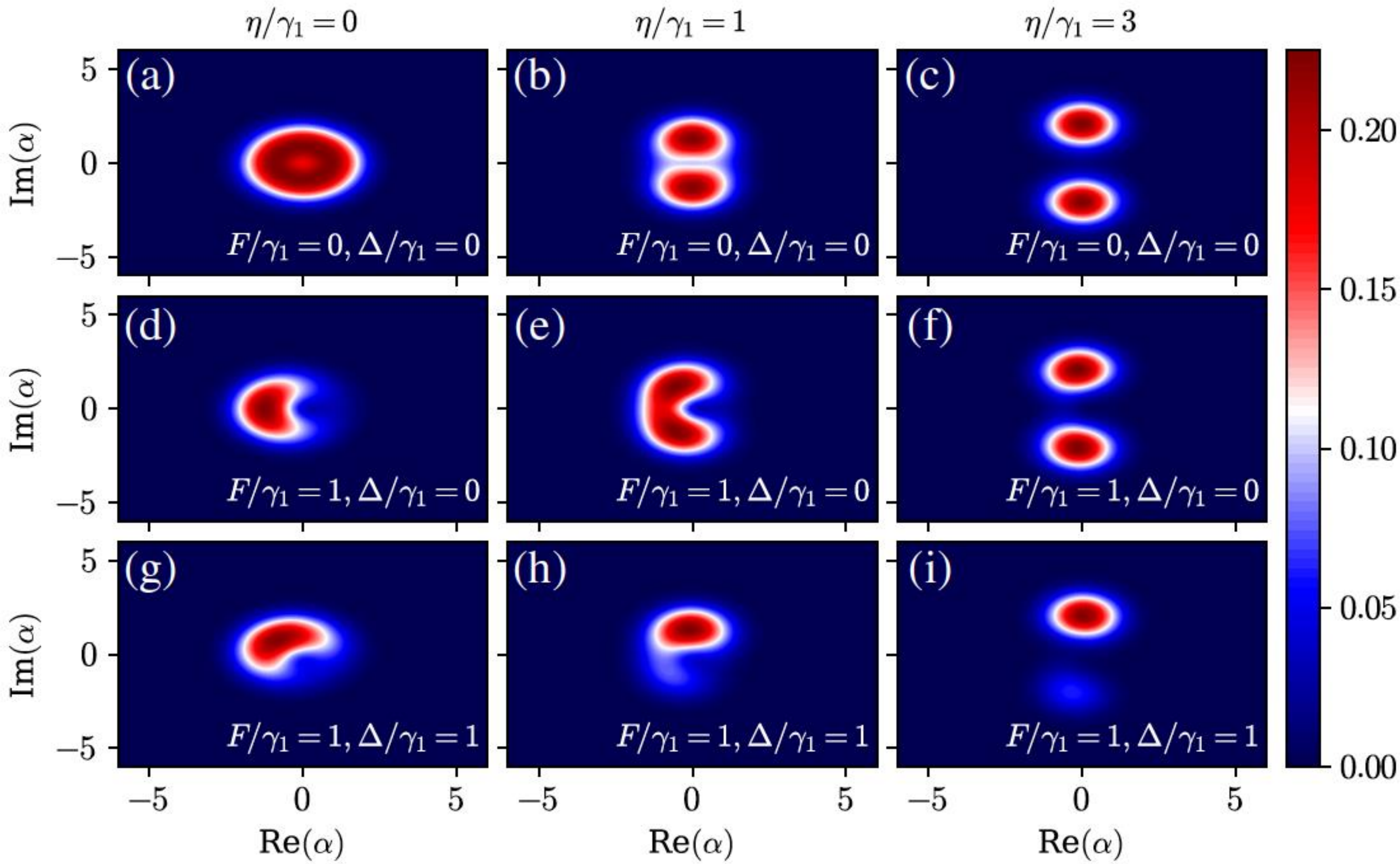




Solid black circle: stable fixed point

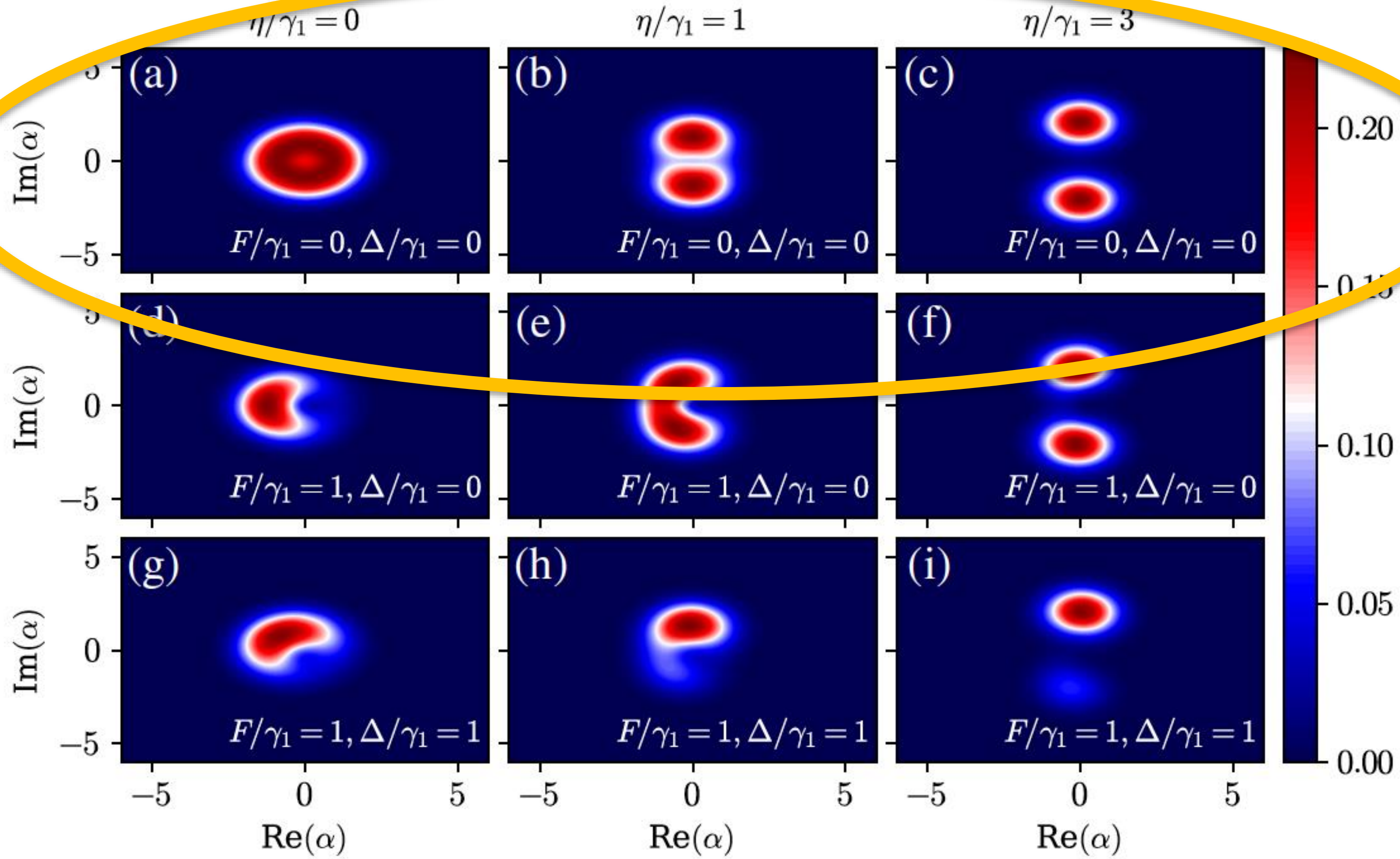
Empty white circle: unstable point

Bifurcation of the Wigner function.



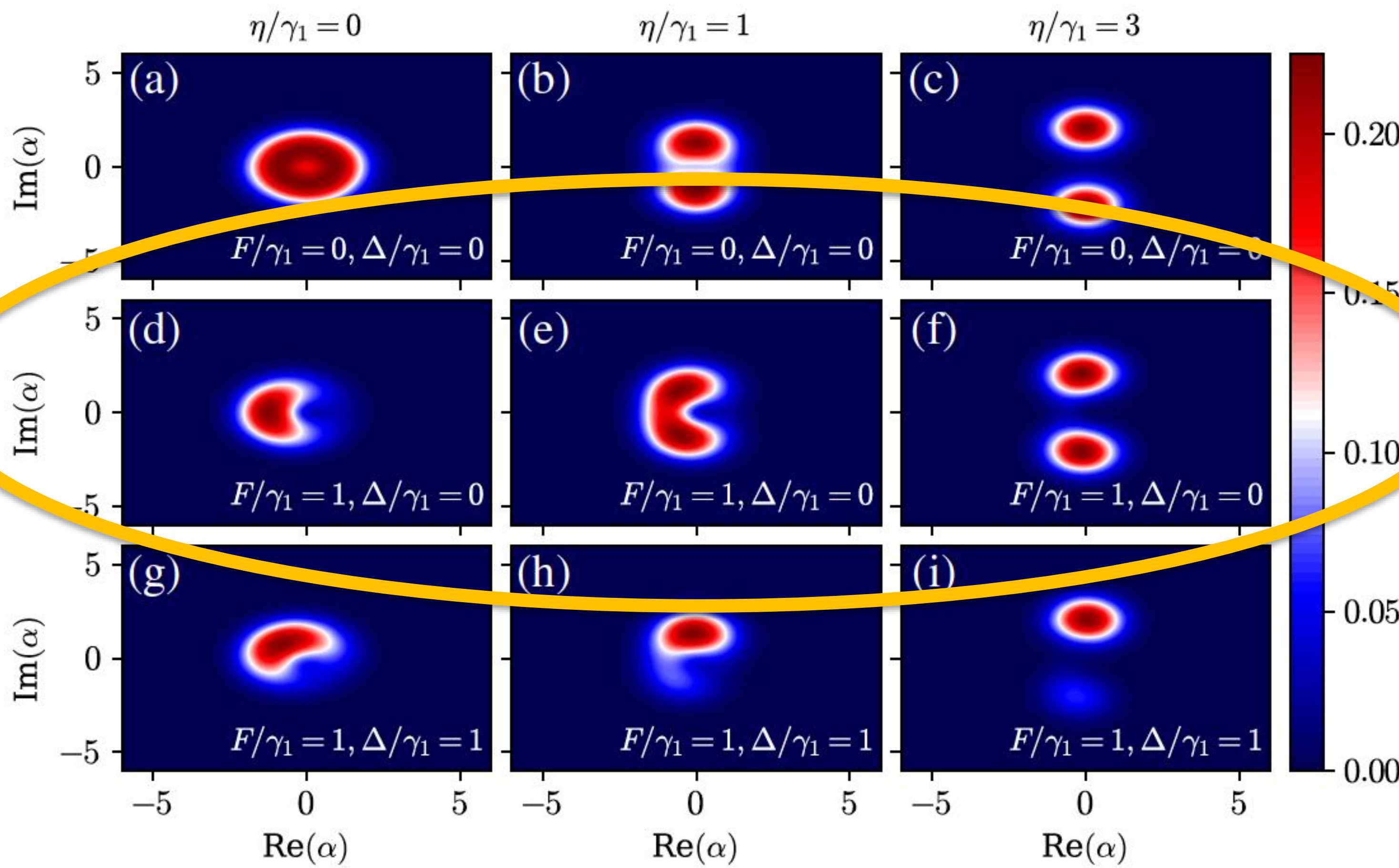
Δ

Bifurcation of the Wigner function.



The bifurcation behavior observed from the classical solutions appears as splitting of the Wigner function into two symmetric parts when driven on resonance. For finite detuning Δ this symmetry is broken as can be seen by the lowering of one of the Wigner function peaks.

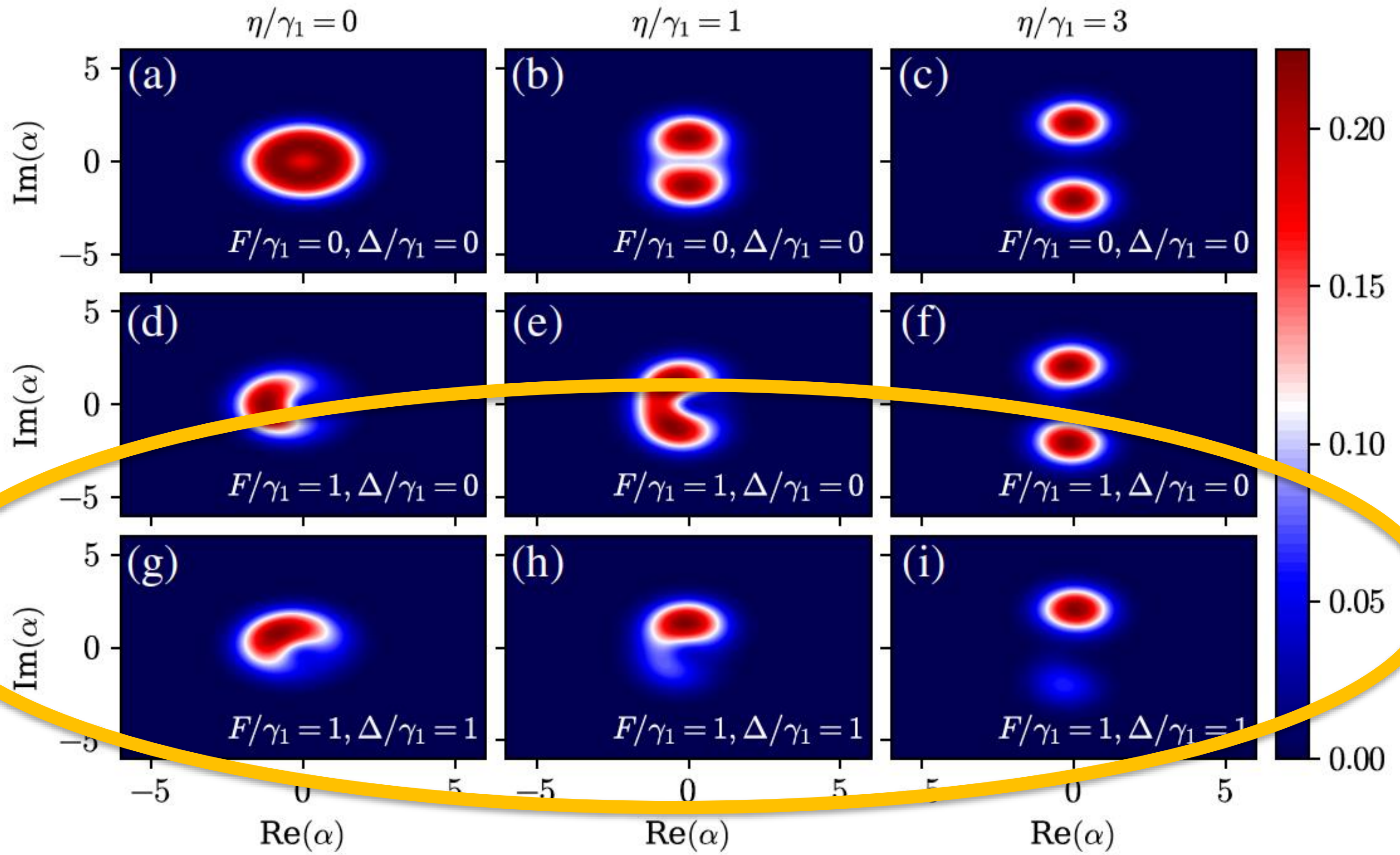
Bifurcation of the Wigner function.



Similar to the undriven case, the oscillator displays symmetric bifurcation with increasing driving

Δ

Bifurcation of the Wigner function.



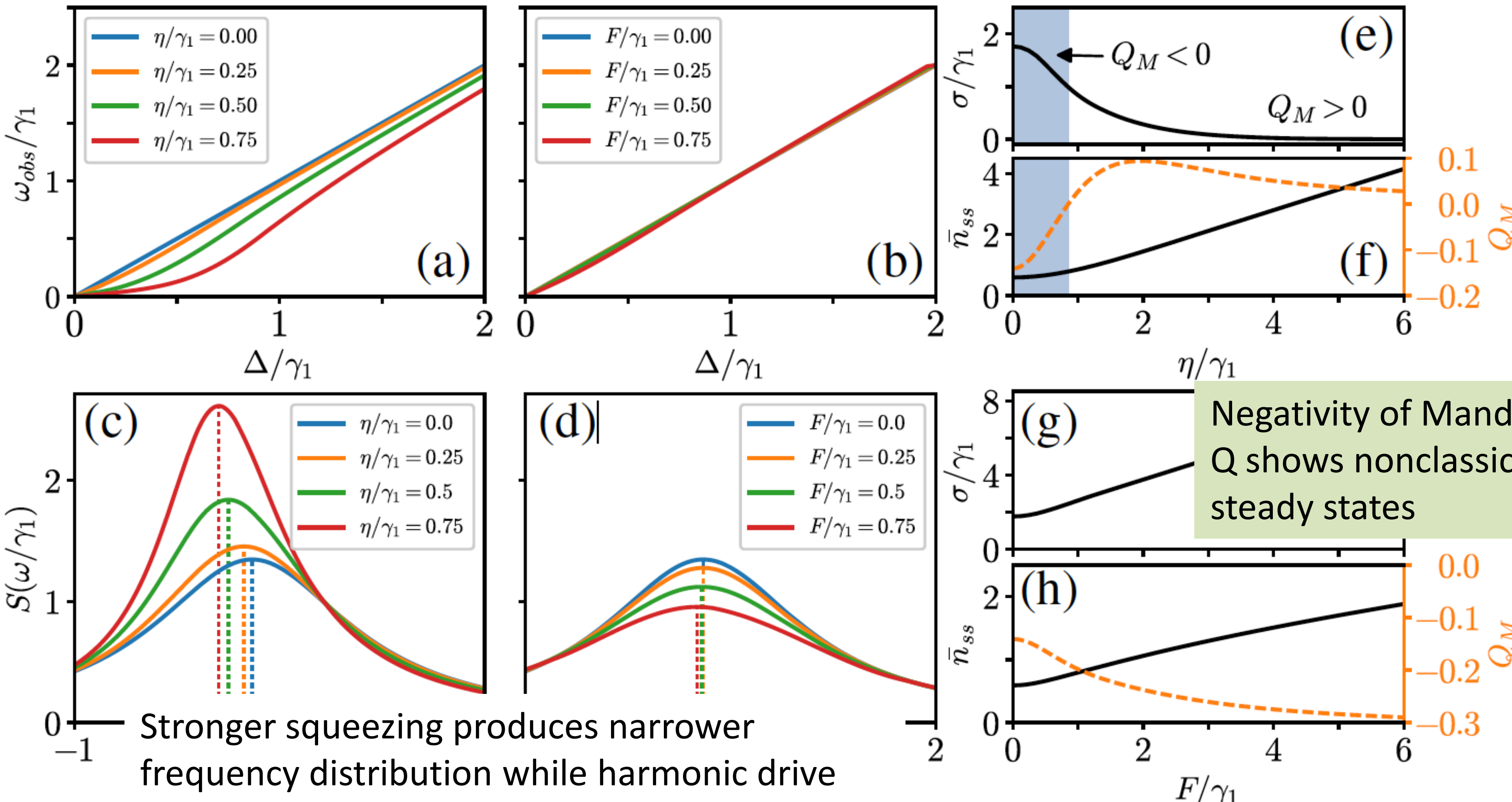
Detuning breaks the symmetry of the lobes in the Wigner function….

Δ

Entrainment of squeezing- and harmonically- driven vdP

Squeezed-driven

Harmonically-driven



Negativity of Mandel Q shows nonclassical steady states

Stronger squeezing produces narrower frequency distribution while harmonic drive has the opposite effect and causes broadening

Mandel $Q_M = \frac{\Delta \hat{n}^2 - \bar{n}}{\bar{n}}$

$$S(\omega) = \int_{-\infty}^{\infty} d\tau \exp(i\omega\tau) \langle \hat{b}^\dagger(\tau)\hat{b}(0)\rangle_{ss}$$

PHYSICAL REVIEW LETTERS 121, 053601 (2018)

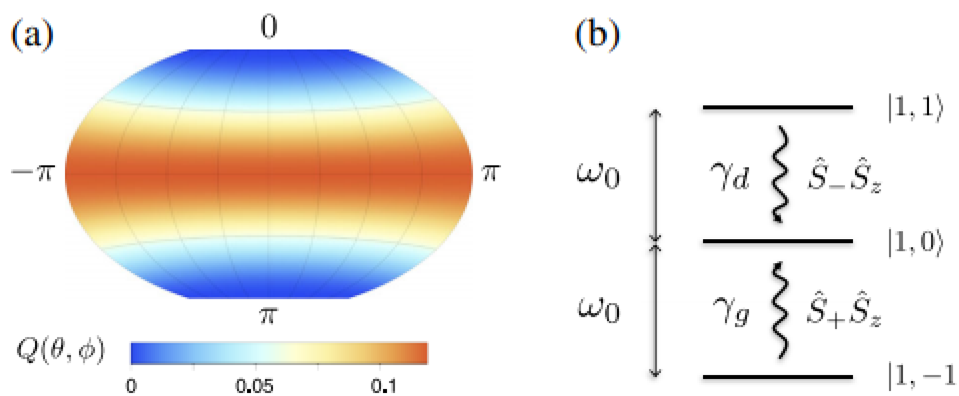
Featured in Physics

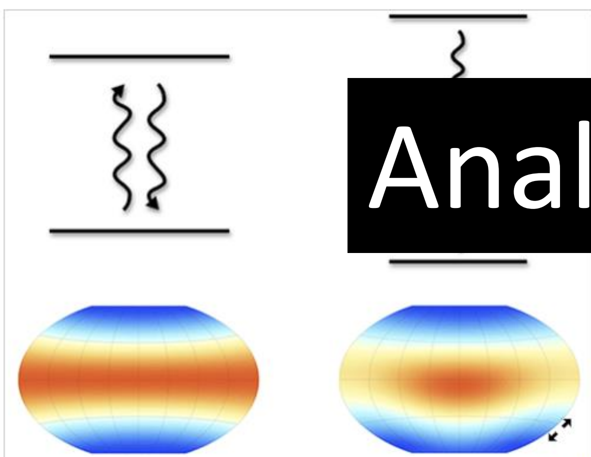
Viewpoint: No Synchronization for Qubits

Leong-Chuan Kwek, Center for Quantum
Institute of Education, Nanyang Technol
International Joint Research Unit, Singa

July 30, 2018 • *Physics* 11, 75

Theorists have determined that a quant
another oscillator





Adapted from A. Roulet and C. Bruder [3]

Analogy to Lasing???

the target state
rojection. The
$= 0$ and $\theta = \pi$
while ϕ specifies the longitude. The ϕ-symmetric distribution centered around the equator is reminiscent of the ring stabilized in the Wigner representation of a Van der Pol limit cycle [6]. (b) The spin-1 ladder, with the eigenstates separated by the natural frequency ω_0. The action of the dissipators is to transfer the populations from the extremal states towards the target state.

See also: ArXiv 1812.10082

Synchronization in two-level quantum systems

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Department of Physics, University of Oslo, NO-0316 Oslo, Norway
(Dated: July 5, 2019)

Recently [1–3], it was shown that dissipative quantum systems with three or more levels are able to synchronize to an external signal, but it was stated that it is not possible for two-level systems as they lack a stable limit cycle in the unperturbed dynamics. At the same time, this statement contradicts [4–7], since synchronization in qubits is numerically demonstrated. However, nothing is said about the appearance of a limit cycle. We show how a quantum two-level system can be understood as containing a valid limit cycle as the starting point of synchronization, and that it can synchronize its dynamics to an external weak signal. This is demonstrated by analytically solving the Lindblad equation of a two-level system immersed in a thermal bath, determining the steady state. This is a mixed state with contributions from many pure states, each of which provides a valid limit cycle. We show that this is sufficient to phase-lock the dynamics to a weak external signal, hence clarifying synchronization in two-level systems. We use the Husimi Q representation to analyze the synchronization region, defining a synchronization measure which characterizes the strength of the phase-locking. Also, we study the stability of the limit cycle and its deformation with the strength of the signal in terms of the components of the Bloch vector of the system. Finally, we generalize the model of the three-level system from [1] in order to illustrate how the stationary fixed point of that model can be changed into a limit cycle similar to the one that we describe for the two-level system.

I. INTRODUCTION

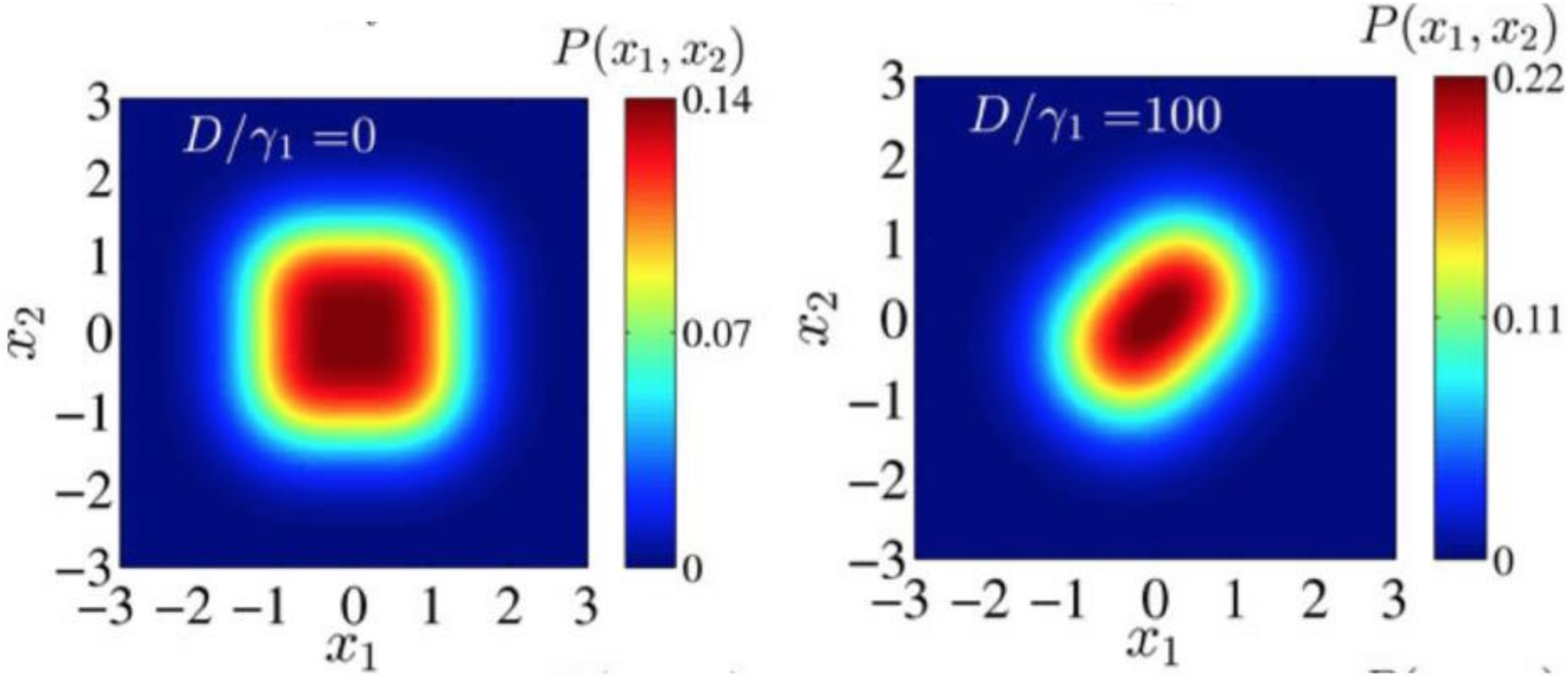
The phenomenon of synchronization occurs in many different situations and has been extensively studied for many years. If an autonomous oscillating system is coupled to another such system or to an external driving force it can synchronize its frequency and phase to the external system. Examples are coupled pendulums, circadian rhythms in living systems or synchronization of fireflies flashing. Common to these systems are the fact that they need to have a stable limit cycle, which means they must be dissipative, so that they can return to the stable cycle after a perturbation, and contain an energy source, so that they can sustain oscillations indefinitely in the presence of dissipation [8].

One of the well studied examples of classical synchronization is the van der Pol oscillator model [8, 9]. Some years ago, the van der Pol model was reformulated in terms of a quantum system [10, 11], and it was shown that when the system is far from the ground state, syn-

signal is possible, although there is no illustration about what is the limit cycle of the system or how it does occur. We show how one can understand the appearance of a valid limit cycle, which is an essential starting point for synchronization, if one aims at relating the quantum version of this phenomena to its classical counterpart. In this context, the system is not completely phase locked, therefore if we accept that the quantum system is similar to a classical system with noise, as is also the case for the van der Pol oscillator [10, 11] (and all quantum systems in general [12–14]), a TLS is in fact capable of synchronization, and the following considerations allow us to understand why.

Our system is immersed in a photon thermal bath so it is able to gain and emit energy, hence creating the dissipating frame synchronization requires. Solving the Lindblad equation for the system in the absence of any external signal one finds that the stationary solutions are mixed states that are constant in time lying on the rotation axis of the Bloch sphere, which we will take to be

Implementation with Circuit QED



Source: J. R. Johansson, G. Johansson, and F. Nori. Optomechanical-like coupling between superconducting resonators. Phys. Rev. A, 90:053833, Nov 2014.;
S. Walter, A. Nunnenkamp, and C. Bruder. Quantum synchronization of two van der pol oscillators. Annalen der Physik, 527(1-2):131{138, 2015.

Unpublished results with Daryl Mok…..

arXiv:1706.04945v1 [quant-ph] 15 Jun 2017

Observing quantum synchronization blockade in circuit quantum electrodynamics

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June 16, 2017

High quality factors, strong nonlinearities, and extensive design flexibility make superconducting circuits an ideal platform to investigate synchronization phenomena deep in the quantum regime. Recently [18], it was predicted that energy quantization and conservation can block the synchronization of two identical, weakly coupled nonlinear self-oscillators. Here we propose a Josephson junction circuit realization of such a system along with a simple homodyne measurement scheme to observe this effect. We also show that at finite detuning, where phase synchronization takes place, the two oscillators are entangled in the steady state as witnessed by the positivity of the logarithmic negativity.

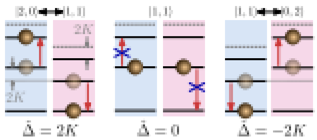

Figure 1: Energetics of quantum synchronization blockade of two weakly coupled anharmonic self-oscillators represented as three level systems with softening anharmonicity $2K$. A resonance between the states $|1,1\rangle$ and $|2,0\rangle$ ($|0,2\rangle$) requires a detuning $\tilde{\Delta} = 2K$ ($\tilde{\Delta} = -2K$) between the oscillators.

1 Introduction

Synchronization of coupled self-sustained oscillating systems is a ubiquitous phenomenon in nature and appears in fields as diverse as biology [1–3], economics [4], sociology [5] and physics, where it was first scientifically described [6]. In the latter field an interesting question is what happens with synchronization in the quantum regime, i.e. when the limit cycle steady states of the oscillators are quantum states with no classical analog. Previous work on quantum synchronization has focused mainly on theoretically identifying and characterizing differences between classical and quantum synchronization [7–18] and on potential applications of the latter [19–21]. Experimental observation of quantum synchronization phenomena is hindered by the stringent requirements of high quantum coherence and strong nonlinearities, both of which are also key requirements for quantum computation.

Driven in large parts by the quest for a quantum computer, superconducting circuits realized with one or multiple Josephson junctions coupled to microwave resonators have become a versatile platform to study light-matter interaction at the single photon level. The design flexibility of superconducting circuits has enabled the realization of a wide range of Hamiltonians [22–24] and quantum reservoirs [25–28] with great precision. This in turn has made possible the observation of textbook nonlinear quantum optics effects taking place in hitherto inaccessible regimes [22, 29].

Here we show that superconducting circuits form an ideal platform for studying synchronization of coupled nonlinear self-sustained oscillators deep in the quantum regime. In particular, we provide the blueprint of a circuit for observing the *quantum synchronization blockade* (QSB) recently predicted by Lörch et al. [18]: In contrast to the classical case where phase synchronization is maximal between two weakly interacting self-sustained oscillators of equal frequencies [30], phase synchronization between two weakly coupled nonlinear self-oscillators, individually stabilized to a Fock state, is suppressed on resonance. Intuition for this effect can be obtained in the perturbative limit of weak interactions [18] as illustrated in Fig. 1: To lowest order, a weak coupling between two nonlinear self-oscillators stabilized to the Fock state $|1\rangle$ can only lead to energy exchange when a finite detuning compensates for the anharmonicity. At zero detuning, energy conservation forbids the exchange of energy to leading order and synchronization is blocked.

At the heart synchronization is a form of correlation. In the quantum regime, the relation between synchronization and entanglement is of particular interest [8, 11, 31, 32]. This relation can also be used to define quantum synchronization: If the correlations present in the synchronized state are non-classical, i.e. if the two oscillators are in an entangled state, then synchronization is of quantum origin. Here we show that when synchronization occurs in our circuit, the steady state of the two oscillators is indeed

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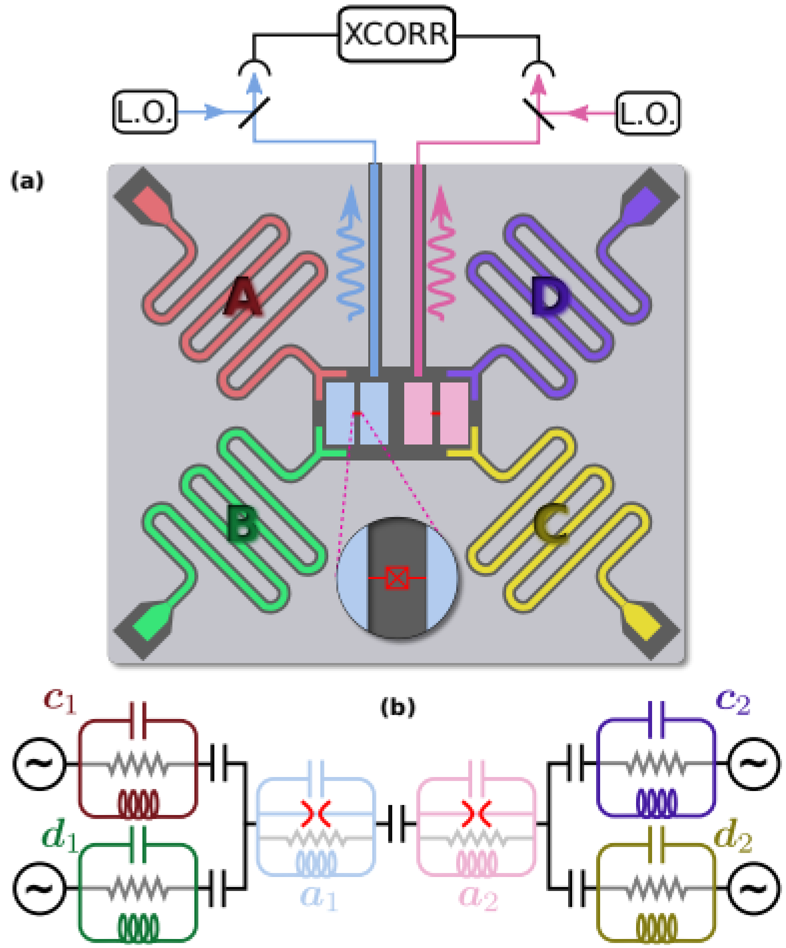

$$H_{\rm disp} = \sum_{j=1}^{2} \left(\Delta_j^a a_j^\dagger a_j + \Delta_j^c c_j^\dagger c_j + \Delta_j^d d_j^\dagger d_j - K_j a_j^\dagger a_j^\dagger a_j a_j - \chi_j^{ac} a_j^\dagger a_j c_j^\dagger c_j - \chi_j^{ad} a_j^\dagger a_j d_j^\dagger d_j \right) + J a_1^\dagger a_1 a_2^\dagger a_2, \quad (1)$$

$$H_{\rm control} = \sum_{j=1}^{2} \left\{ \varepsilon_j^a \left(a_j + a_j^\dagger \right) + \varepsilon_j^c \left(c_j + c_j^\dagger \right) + \varepsilon_j^d \left(d_j + d_j^\dagger \right) \right\}. \quad (2)$$

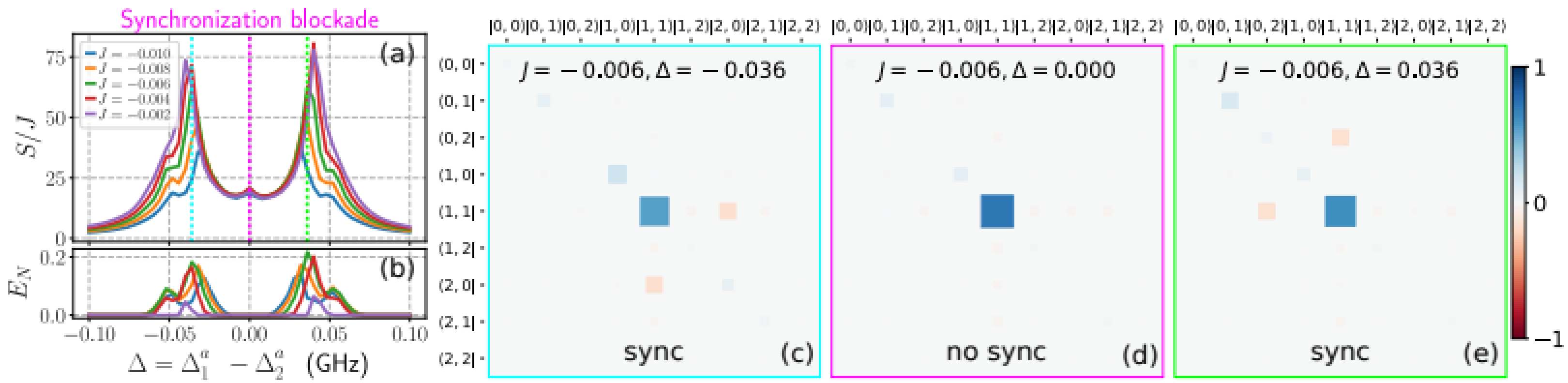

Figure 4: **(a)** Normalized quantum synchronization measure S/J as a function of the bare detuning $\Delta = \Delta_1^a - \Delta_2^a$ between the two nonlinear self-oscillators for different values of the bare inter-oscillator dispersive coupling strength J. Δ_2^a is varied while Δ_1^a is kept fixed. The steady state is obtained from a quantum trajectory simulation by averaging the long time (i.e. $t \gg 1/\gamma_\pm$) temporal averages over 500 trajectories. **(b)** Logarithmic negativity $E_N = \log_2 ||\boldsymbol{\rho}_{ss}^{\mathrm{PT}}||_1$, showing that when the oscillators synchronize, entanglement is present, i.e. $E_N > 0$. **(c)** to **(e)** Hinton diagrams at three different values of Δ

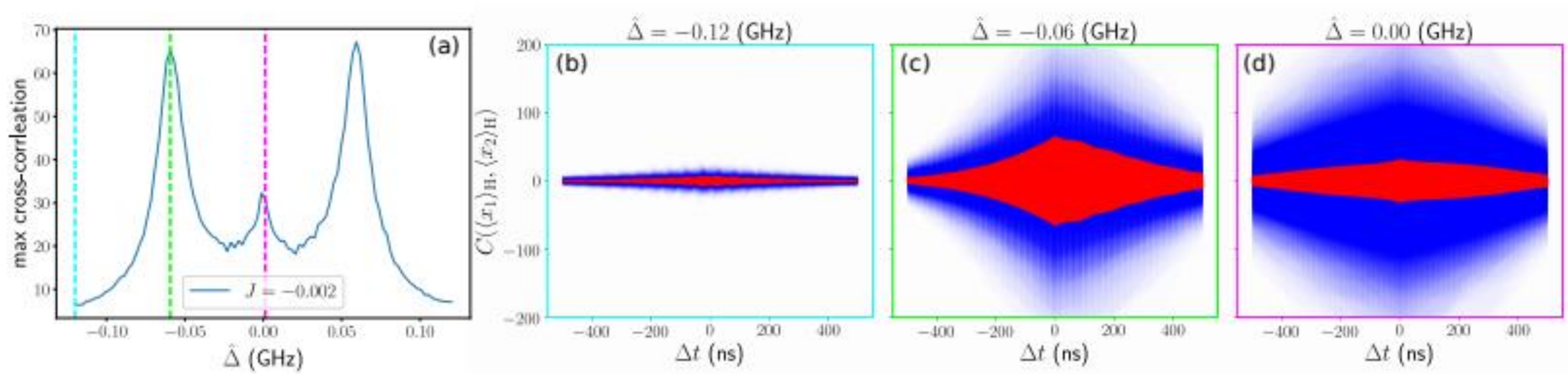

Figure 5: **(a)** Maximum of the averaged cross-correlated homodyne signals as a function of the renormalized detuning $\hat{\Delta}$ between the nonlinear oscillators. Clearly the cross-correlation mirrors the synchronization measure of Fig. 4 (a). **(b)** to **(d)** Cross-correlation functions of the homodyne signals corresponding to the three different values of $\hat{\Delta}$ marked by dashed vertical lines in (a). The individual trajectories are shown in blue and the average over 1000 trajectories is shown in red. Parameter values are the same as in Figs. 3 and 4.

ly regulated by ligands, evidenced by the opposing roles of Jagged1 and DLL4 in angiogenesis and the role of the Notch1-DLL4 axis in lymphatic development and the lymphatic system (*45*, *46*). Certain advantages of receptor-specific therapeutics have begun to emerge in studies describing the inhibition of tumor growth by Notch1-antagonist antibodies and the control of graft-versus-host disease by DLL1/4-antagonist antibodies (*47*, *48*). The Notch1(11–13)–DLL4$_{SLP}$(N-EGF1) structure provides new insights toward the development of receptor- and ligand-specific drugs. Antibodies against Notch EGF11 and 12 reportedly have limited effectiveness in cellular assays; based on the structure, this is likely due to occlusion of relevant protein epitopes by the Ser^{435} O-glucose and Thr^{466} O-fucose moieties (*49*). More effective therapeutic targeting of EGF11 and 12 may be achieved with engineered high-affinity ligands such as those presented here, which have co-evolved with Notch receptors to accommodate O-glycan binding.

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REPORTS

QUANTUM ENGINEERING

Confining the state of light to a quantum manifold by engineered two-photon loss

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Physical systems usually exhibit quantum behavior, such as superpositions and entanglement, only when they are sufficiently decoupled from a lossy environment. Paradoxically, a specially engineered interaction with the environment can become a resource for the generation and protection of quantum states. This notion can be generalized to the confinement of a system into a manifold of quantum states, consisting of all coherent superpositions of multiple stable steady states. We have confined the state of a superconducting resonator to the quantum manifold spanned by two coherent states of opposite phases and have observed a Schrödinger cat state spontaneously squeeze out of vacuum before decaying into a classical mixture. This experiment points toward robustly encoding quantum information in multidimensional steady-state manifolds.

Stabilizing the state of a system in the vicinity of a predefined state despite the presence of external perturbations plays a central role in science and engineering. On a quantum system, stabilization is a fundamentally more subtle process than on a classical system, as it requires an interaction that, quantum mechanically, is always invasive. The mere act of learning something about a system perturbs it. Carefully designed nondestructive quantum measurements have recently

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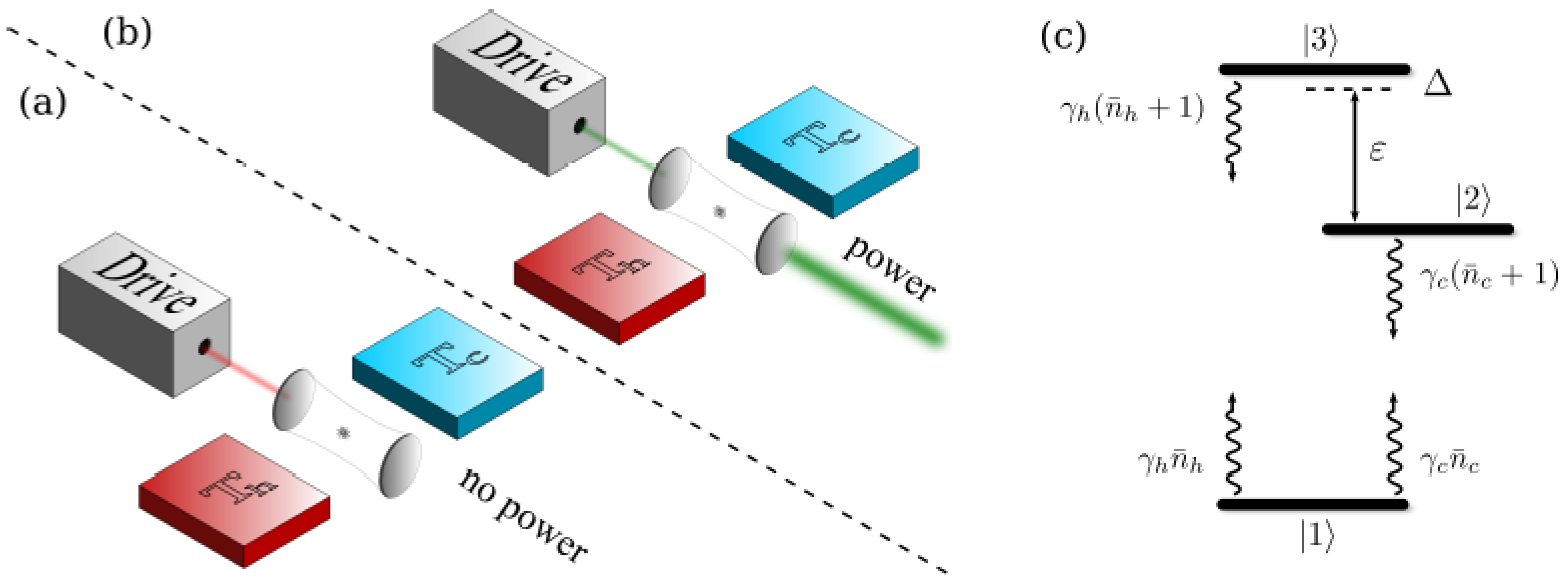

FIG. 1. A quantum thermal machine can be used to observe quantum synchronisation. The engine consists of a three level maser driven three-level atom which generates output power when coupled to two dissimilar baths. The relationship between power, detuning and the driving strength is understood as arising from the synchronization of the three-level atom to the external drive. At a given driving strength, if the driving field is far detuned from the relevant maser transition, the system cannot synchronise to the external drive. The output power is very low as seen in (a). If the detuning is in the synchronisation region, the engine power is reinforced by synchronisation as depicted in (b). (c) Transition between levels $|2\rangle$ and $|3\rangle$ is coupled by a coherent field of strength ε. Levels $|1\rangle$-$|3\rangle$ are coupled by a hot bath at temperature T_h while levels $|2\rangle$-$|3\rangle$ are coupled by a cold bath at temperature T_c.

See: arXiv 1812.10082

Power Arnauld Tongue

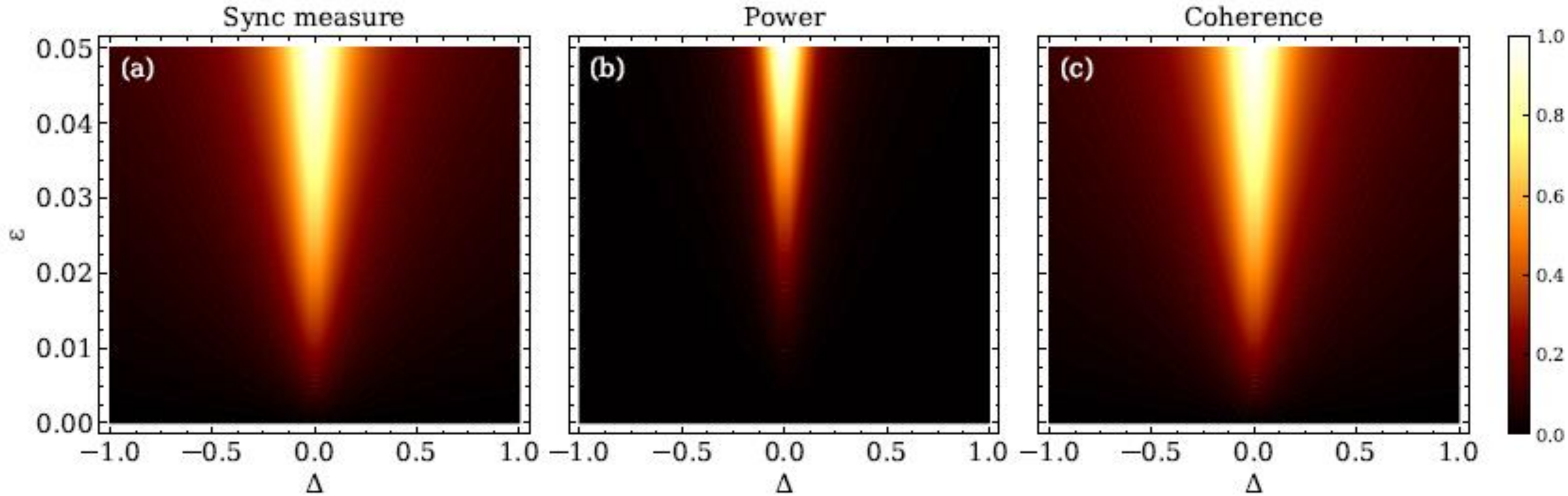

FIG. 2. Contours for (a) the measure of synchronization S, (b) the corresponding power of a thermal engine $-P$, and (c) the corresponding measure of coherence $\mathcal{C}$ are plotted against the detuning Δ on the x-axis and the strength of the drive ε on the y-axis. The parameters are fixed as $\gamma_h = 0.01$, $\bar{n}_h = 5$, $\bar{n}_c = 10^{-3}$, $\gamma_c = 10\gamma_h$, and $\omega_3 - \omega_2 = 10/\gamma_h$.

$$\mathcal{S}(\varphi_1, \varphi_2) = \int d\Omega \; Q(\theta, \xi, \varphi_1, \varphi_2) - \frac{1}{4\pi^2}$$

Summary

- We review classical and quantum synchronization
- We consider a driven self-sustained oscillator with a squeezing Hamiltonian instead of a harmonic drive.
- We analyze this system in the classical and quantum regime.
- We demonstrate that strong entrainment is possible for small values of squeezing, and in this regime, the states are nonclassical.
- We show that the quality of synchronization measured by the FWHM of the power spectrum is enhanced with squeezing.

Thank you very much