Центр
Квантовых
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Physics Faculty
Lomonosov MSU

Quantum state engineering and tomography with high-dimensional spatial states of photons

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Outline

Quantum state engineering in SPDC with shaped pump

Quantum state tomography of spatial states

- Adaptive quantum tomography of high-dimensional states
- Fighting SPAM errors with neural networks
- Experimental shadow tomography

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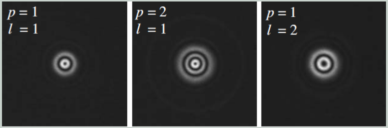
Paraxial spatial modes

- In polar coordinates $\{k_\perp, \phi\} \Longrightarrow$ - Laguerre-Gaussian modes
- In cartesian coordinates $\{k_x, k_y\} \Longrightarrow$ - Hermite-Gaussian modes

Laguerre-Gaussian modes

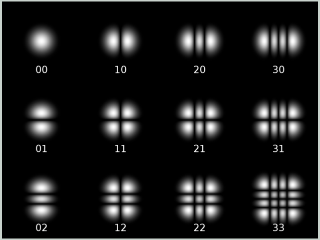
$$\varphi_{pl}(k_\perp, \phi) \propto L_p^{|l|}\left(\frac{k_\perp^2}{2(\Delta k_\perp)^2}\right)\exp\left(-\frac{k_\perp^2}{4(\Delta k_\perp)^2}\right) \times \exp\left(il\phi + i\left(p - \frac{|l|}{2}\right)\pi\right)$$

- l - topological charge of the beam
- LG beam carries orbital angular momentum of $l\hbar$ per photon

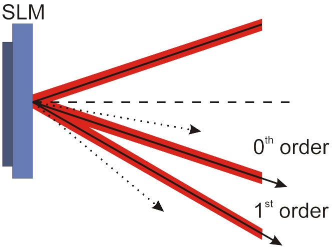


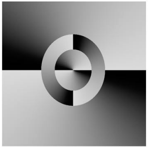
Hermite-Gaussian modes

$$\varphi_{nm}(k_x, k_y) \propto H_n\left(\frac{k_x^2}{(\Delta k_x)^2}\right) H_m\left(\frac{k_y^2}{(\Delta k_y)^2}\right)\exp\left(-\frac{k_x^2+k_y^2}{4(\Delta k_\perp)^2}\right)$$



A phase-only SLM is used to display holograms

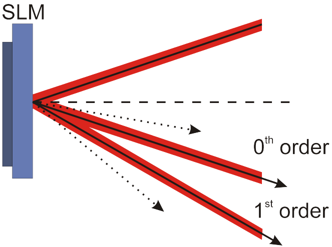


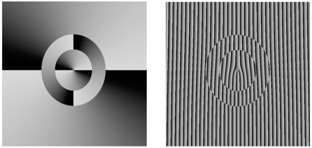


Options:

- Zeroth order, phase-only modulation

A phase-only SLM is used to display holograms

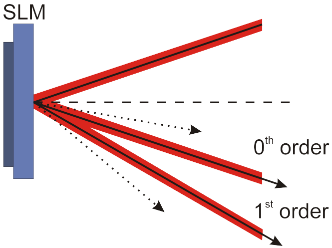


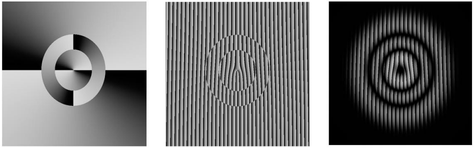


Options:

- Zeroth order, phase-only modulation
- First order, phase-only modulation

A phase-only SLM is used to display holograms

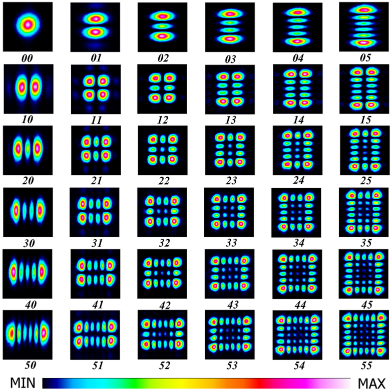


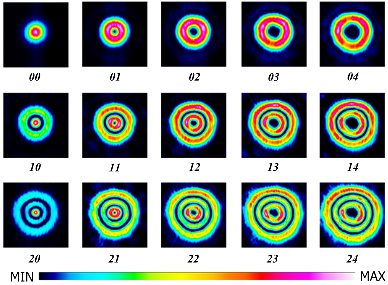


Options:

- Zeroth order, phase-only modulation
- First order, phase-only modulation
- First order, phase and amplitude modulation

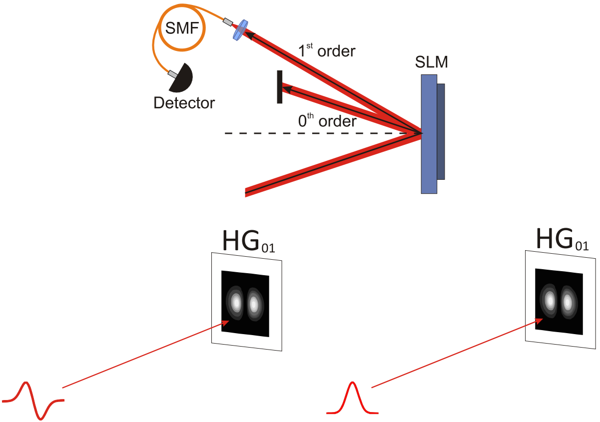
Experimentally obtained far-field distributions





Spatial mode filter

Idea: "reverse" the generation setup and use same holograms as filters



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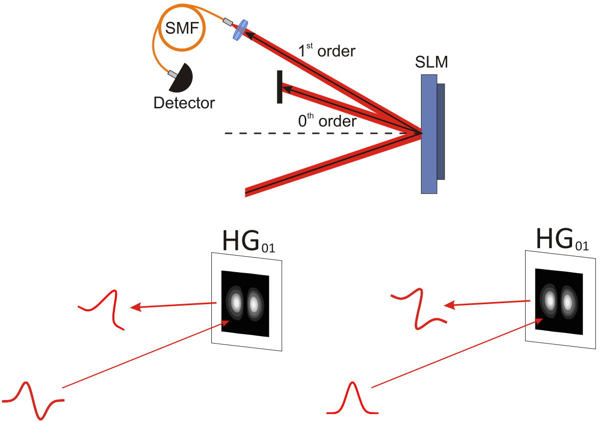

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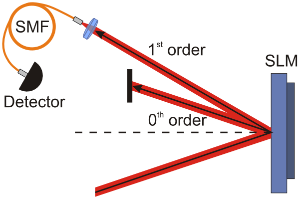
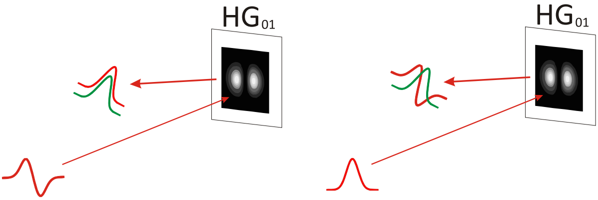




I.B. Bobrov et al. Optics Express **23**, 649 (2015)

Schmidt mode decomposition

SPDC biphoton state:

$$|\psi\rangle = |vac\rangle + \mathrm{const} \times \int d\vec{k}_s d\vec{k}_i \Psi(\vec{k}_s, \vec{k}_i) |1\rangle_s |1\rangle_i,$$

$\vec{k}_s$, $\vec{k}_i$ – wavevectors of signal and idler photons
$|1\rangle_s$, $|1\rangle_i$ – single-photon Fock states of the corresponding modes.

$$\Psi(\vec{k}_{s_\perp}, \vec{k}_{i_\perp}) = \sum_{n=0}^{\infty} \sqrt{\lambda_n} \psi_n(\vec{k}_{s_\perp}) \chi_n(\vec{k}_{i_\perp}).$$

Schmidt number:

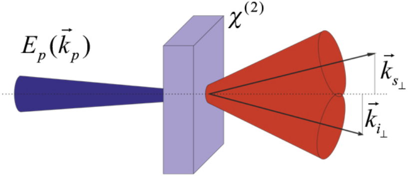
$$K = \frac{1}{\sum_{n=0}^{\infty} \lambda_n^2}$$

Spatial two-photon amplitude

$$\Psi(\vec{k_{s\perp}}, \vec{k_{i\perp}}) \propto E_p(\vec{k_{s\perp}} + \vec{k_{i\perp}})\mathrm{sinc}\left(\frac{L(\vec{k}_{s_\perp} - \vec{k}_{i_\perp})^2}{4k_p}\right).$$

$$K = \left(\frac{a^2 + b^2}{2ab}\right)^2,$$

a – pump beam width
b – phase-matching bandwidth



C. Law and J. Eberly, Phys. Rev. Lett. 92, 127903 (2004)

Spatial two-photon amplitude

$$\Psi(\vec{k_{s\perp}}, \vec{k_{i\perp}}) \propto E_p(\vec{k_{s\perp}} + \vec{k_{i\perp}})\mathrm{sinc}\left(\frac{L(\vec{k}_{s_\perp} - \vec{k}_{i_\perp})^2}{4k_p}\right).$$

$$\Psi(\vec{k_{s\perp}}, \vec{k_{i\perp}}) \propto \exp\left(-a^2\frac{(\vec{k}_{s_\perp} + \vec{k}_{i_\perp})^2}{2}\right)\exp\left(-b^2\frac{(\vec{k}_{s_\perp} - \vec{k}_{i_\perp})^2}{2}\right),$$

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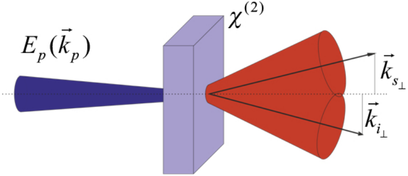

C. Law and J. Eberly, Phys. Rev. Lett. 92, 127903 (2004)

Decomposition in terms of HG modes

- Biphoton wavefunction:
$$\Psi_\perp\left(\vec{k}_{1_\perp},\vec{k}_{2_\perp}\right)\propto E_p^*\left(\frac{\vec{k}_{1_\perp}+\vec{k}_{2_\perp}}{2}\right)\mathrm{sinc}\left[C\left(\vec{k}_{1_\perp}-\vec{k}_{2_\perp}\right)^2\right]$$
- Let the pump mode be Hermite-Gaussian:

$$E_p(k_x,k_y)=HG_{nm}(k_x,k_y)=HG_n(k_x)HG_m(k_y)$$

$$HG_n(k)\propto\sqrt{w}H_n(ak)\exp\left(-\frac{a^2k^2}{2}\right)$$

- Biphoton state may be decomposed as[2]:

$$|\psi_{nm}\rangle=\sum_{j,k,s,t}C_{jkst}^{(nm)}\left|HG_{jk}(k_{1x},k_{1y})\right\rangle\left|HG_{st}(k_{2x},k_{2y})\right\rangle$$

[2] S.P.Walborn and A.H. Pimentel, Journal of Physics 45(16):165502 (2012)

Decomposition in terms of HG modes

- Biphoton wavefunction:
  $$\Psi_\perp\left(\vec{k}_{1\perp}, \vec{k}_{2\perp}\right) \approx E_p^*\left(\frac{\vec{k}_{1\perp}+\vec{k}_{2\perp}}{2}\right) \exp\left(-\frac{b^2(\vec{k}_\perp-\vec{k}'_\perp)^2}{2}\right)$$
- Let the pump mode be Hermite-Gaussian:

$$E_p(k_x, k_y) = HG_{nm}(k_x, k_y) = HG_n(k_x)HG_m(k_y)$$

$$HG_n(k) \propto \sqrt{w}H_n(ak)\exp(-\frac{a^2k^2}{2})$$

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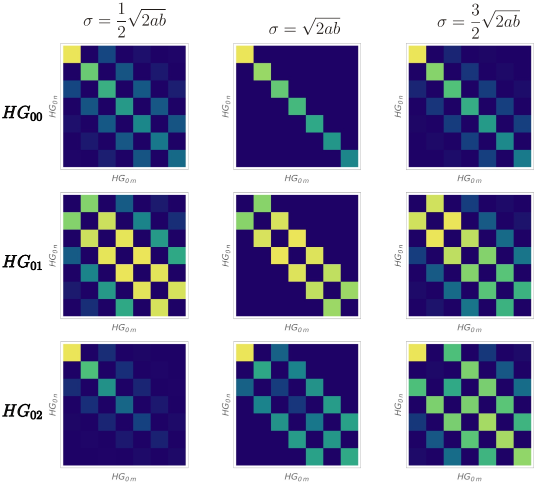
[2] S.P.Walborn and A.H. Pimentel, Journal of Physics 45(16):165502 (2012)

Hermite-Gaussian pump[3], Schmidt number $K = 6$



[3] S.P.Walborn and A.H. Pimentel, Journal of Physics 45(16):165502 (2012)

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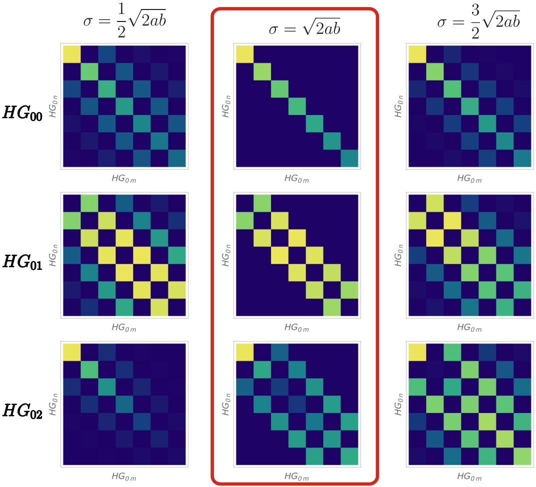
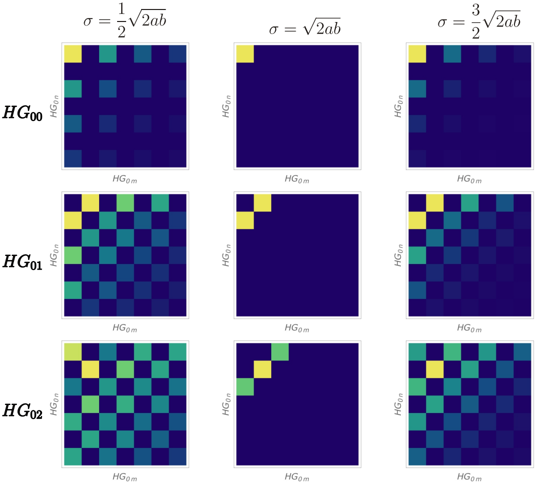


[3] S.P.Walborn and A.H. Pimentel, Journal of Physics 45(16):165502 (2012)

Hermite-Gaussian pump[4], Schmidt number $K = 1$



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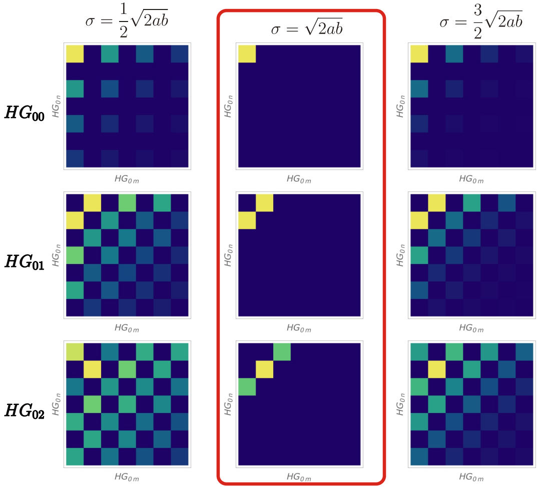
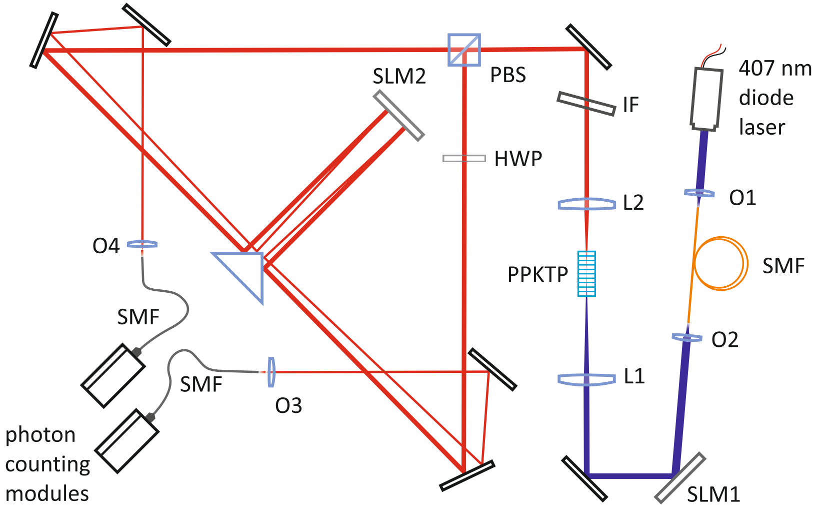
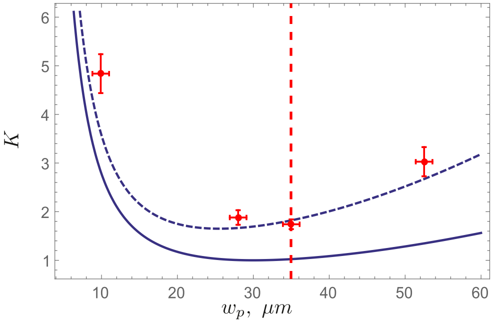


[4] S.P.Walborn and A.H. Pimentel, Journal of Physics 45(16):165502 (2012)

Experimental setup



Experimental results: Schmidt number

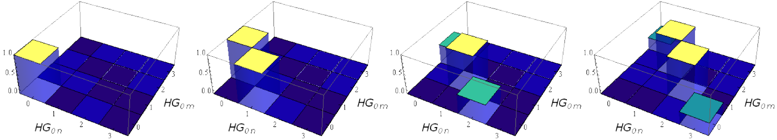


$$K = \left(\frac{w^2 + \delta^2}{2w\delta}\right)^2 \to \beta\left(\frac{w^2 + \alpha^2\delta^2}{2w\alpha\delta}\right)^2 *$$

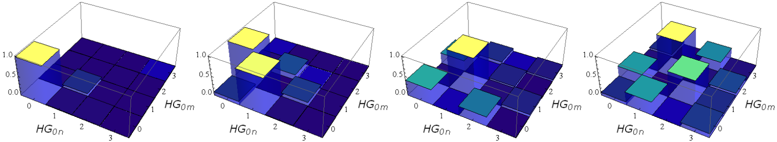
$$\alpha = 0.85, \ \beta = 1.65$$

*F. Miatto, H. Pires, S. Barnett, and M. van Exter, The European Physical Journal D 66, 1 (2012).

Results for non-Gaussian pump

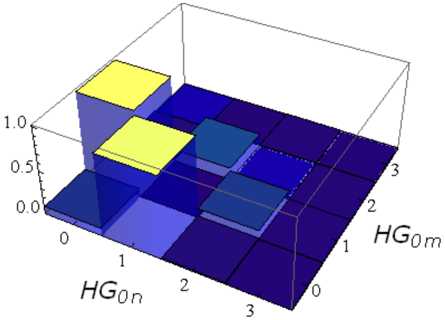


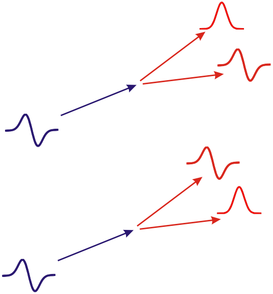
Theoretical distributions for $|C^{(0)}_{nm}|^2$, $|C^{(1)}_{nm}|^2$, $|C^{(2)}_{nm}|^2$, $|C^{(3)}_{nm}|^2$.



Measured distributions for $|C^{(0)}_{nm}|^2$, $|C^{(1)}_{nm}|^2$, $|C^{(2)}_{nm}|^2$, $|C^{(3)}_{nm}|^2$.

Spatial Bell state





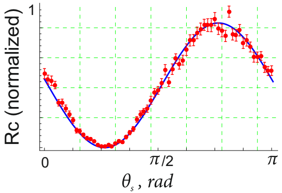
Measured $|C_{nm}^{(1)}|^2$ for HG_{01} pump

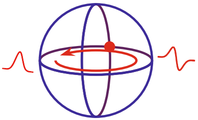
$$|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|\mathrm{HG}_{00},\mathrm{HG}_{01}\rangle + |\mathrm{HG}_{01},\mathrm{HG}_{00}\rangle)$$

E.V.Kovlakov et al. Phys. Rev. Lett. **118**, 030503 (2017)

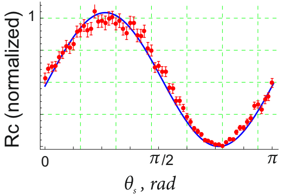
$$\text{Detection modes: } |\theta_{s,i}\rangle = \cos(\frac{\theta_{s,i}}{2})\,|\mathrm{HG}_{00}\rangle + \sin(\frac{\theta_{s,i}}{2})\,|\mathrm{HG}_{01}\rangle$$

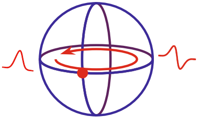
$$R_c(\theta_s, \theta_i) \propto |\,\langle\theta_s|\,\langle\theta_i|\,|\Psi^+\rangle\,|^2 \propto \sin(\theta_s - \theta_i)$$





$\theta_i = \pi/4$

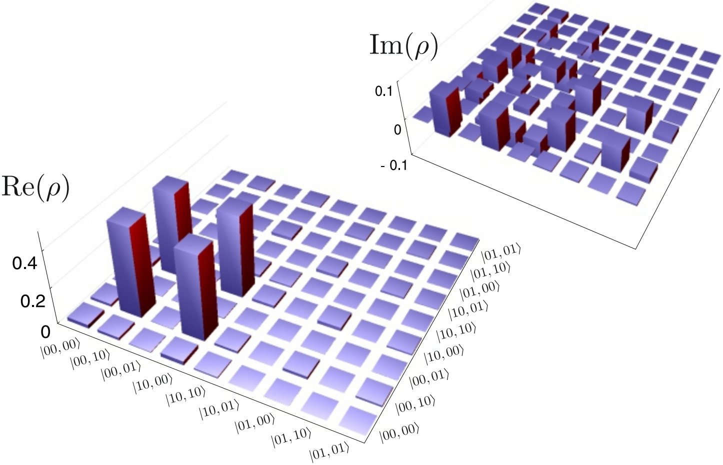




$\theta_i = 3\pi/4$

Measured CHSH inequality violation: $S = (2.81 \pm 0.05) > 2.$

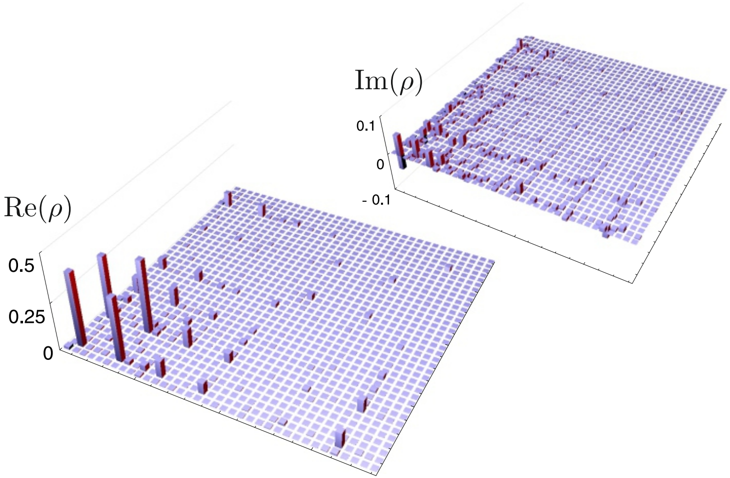
Full state tomography in larger space



Fidelity with an ideal Bell state is 0.97

E.V.Kovlakov et al. Phys. Rev. Lett. **118**, 030503 (2017)

Full state tomography in larger space



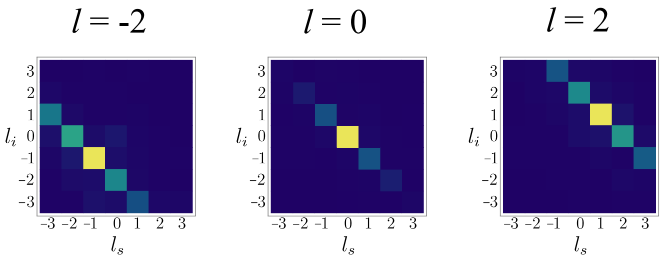
Fidelity with an ideal Bell state is 0.72

E.V.Kovlakov et al. Phys. Rev. Lett. **118**, 030503 (2017)

OAM-carrying modes LG_{0l} are easier to deal with

Consider a pump beam with $E_p = \mathrm{LG}_{0l}(\rho, \varphi)$

OAM conservation rule: $l = l_s + l_i$



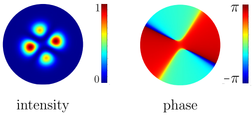
An idea is to take a superposition of pump modes with different l

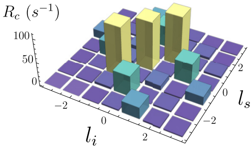
$$E_p = \sum_l \alpha_l \mathrm{LG}_{0l}(\rho, \varphi)$$

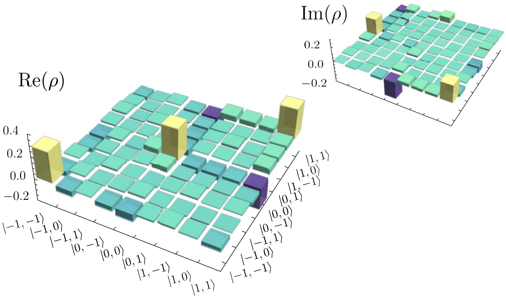
Coefficients are obtained via an optimization procedure

Target state: $|\psi\rangle = \frac{1}{\sqrt{3}}\left(e^{i\theta_1}|-1,-1\rangle + |0,0\rangle + e^{i\theta_2}|1,1\rangle\right)$

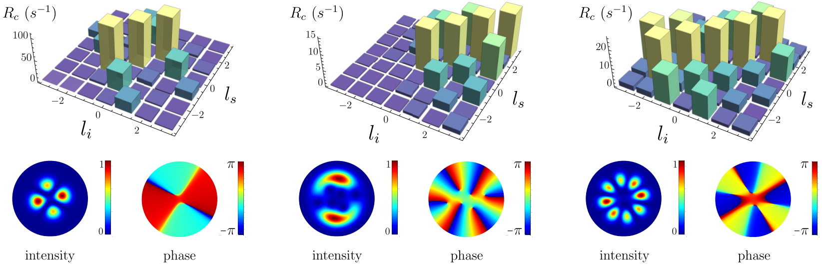
$$\alpha_{-2} = 0.76 - 0.11i, \quad \alpha_0 = -0.12 + 0.15i, \quad \alpha_2 = 0.30 - 0.53i$$







Full control over the state space dimensionality



Central picture corresponds to a "vortex pancake" pump beam (J.P.Torres et al. Phys. Rev. A 67, 052313 (2003)

E.V.Kovlakov, S.S.Straupe, S.P.Kulik, Phys. Rev. A **98**, 060301(R) (2018)
Independent work: S.-L.Liu et al. Phys. Rev. A 98, 062316 (2018)

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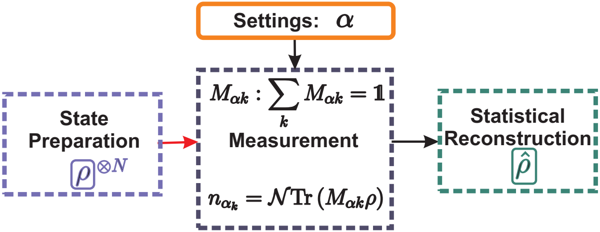
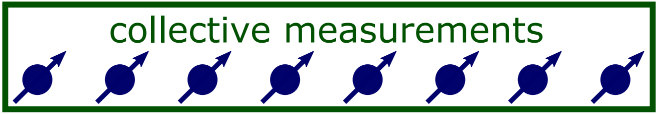
Quantum state tomography



- **State:** $\rho \in \mathcal{H}$: $(d^2 - 1)$ real parameters, N copies
- **Measurements:** $\mathbb{M}_\alpha : \{M_{\alpha k} : \sum_k M_{\alpha k} = \mathbb{1}\}$ – POVM
- **Born's rule:** $\mathrm{P}(k|\rho, \alpha) = \mathrm{Tr}(M_{\alpha k}\rho)$ – k-th outcome probability
- **Experimental data:** $\mathcal{D} : \{n_{\alpha k}\}$ – outcomes, obtained for the configuration α

One should estimate $\hat{\rho}$ using data $\mathcal{D}$

Fidelity: $F(\rho, \hat{\rho}) = \left[\mathrm{Tr}\sqrt{\sqrt{\rho}\hat{\rho}\sqrt{\rho}}\right]^2$



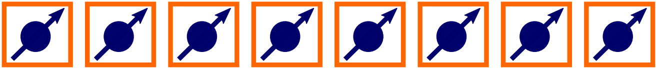
Ultimate bound for pure states of qubits and **collective measurements** (Massar-Popescu):

$$1 - F \geq \frac{1}{N+2}$$

How much data do we need?

Fidelity: $F(\rho, \hat{\rho}) = \left[\mathrm{Tr}\sqrt{\sqrt{\rho}\hat{\rho}\sqrt{\rho}}\right]^2$

individual measurements

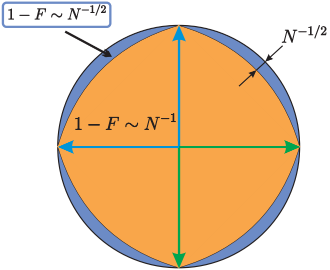


Ultimate bound for pure states of qubits and **individual measurements** (Gill-Massar):

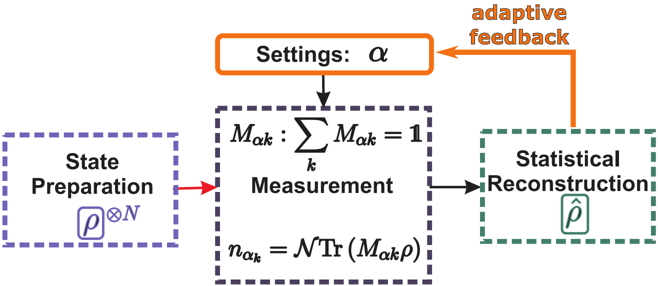
$$1 - F \geq \frac{9}{4} N^{-1}$$

Is it possible to achieve the GM bound?

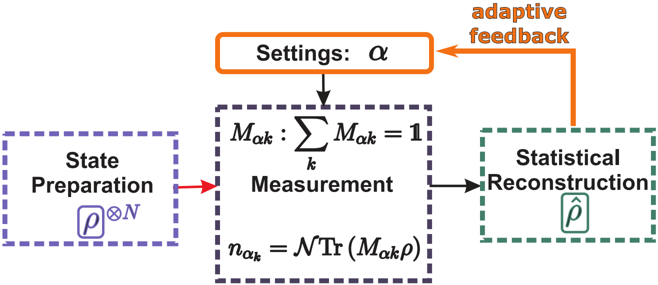
- Fidelity: $F(\rho, \hat{\rho}) = \left[\mathrm{Tr}\sqrt{\sqrt{\rho}\hat{\rho}\sqrt{\rho}}\right]^2$
- Infidelity for *pure states* scales as $1/\sqrt{N}$ for almost all projective measurments[6]



[6] D.H.Mahler, *et al.* Phys. Rev. Lett. **111**, 183601 (2013)



Optimal measurement: $\alpha_{n+1} = \arg\max_{\alpha} \sum_{\gamma_n} p(\gamma_n|\alpha) U(\alpha, \mathcal{D}_n)$

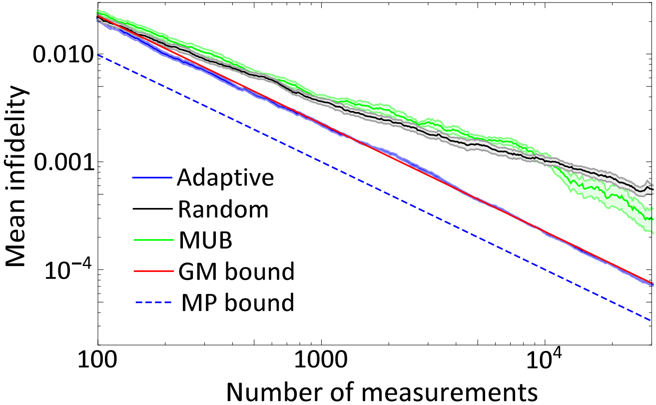


Optimal measurement [a]:

$$\alpha_{n+1} = \arg\max_{\alpha} \left[\mathcal{H}(\pi_n(\rho|\mathcal{D}_n)) - \mathbb{E}_{\pi_n(k|M_\alpha,D)}\left(\mathcal{H}\left[\pi_{n+1}(\rho|k,\alpha,\mathcal{D}_n)\right]\right)\right]$$

[a] F. Huszár and N. M. T. Houlsby, Phys. Rev. A **85**, 052120 (2012)

Experimental qubit tomography



MUB *worst case*: $1 - F \sim 1/\sqrt{N}$

MUB *best case* (eigenbasis): $1 - F \sim 1/N$

Adaptive tomography[7] *always* gives $1 - F \sim 1/N$

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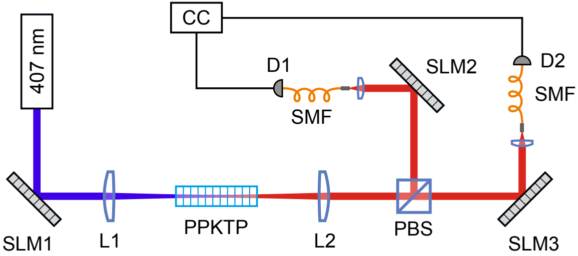
[7] K.S.Kravtsov, *et al.*, Phys. Rev. A **87**, 062122 (2013)

Problems with high-dimensional states

A typical setup looks like this:

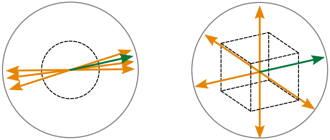


- Bipartite scenario: measurements are **factorized** $M = M_A \otimes M_B$
- Bayesian experimental design is **numerically intractable** in high dimensions

We need a simpler approach!

State-orthogonal measurements

What do all adaptive protocols have in common?



Measurements tend to align with the true state or orthogonal ones

Heuristic: low probability outcomes are of most importance

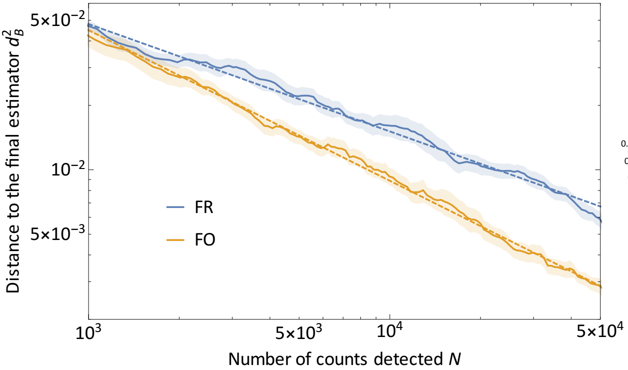
A measurement M is **orthogonal** to the state ρ, if

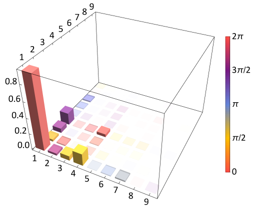
$$\mathrm{Tr}(M\rho) = 0$$

The protocol should contain measurements M, which are orthogonal to all eigenvectors $|\psi_k\rangle$ of the true state with nonzero eigenvalues:

$$M|\psi_k\rangle = 0$$

Gaussian state, $D = 9$



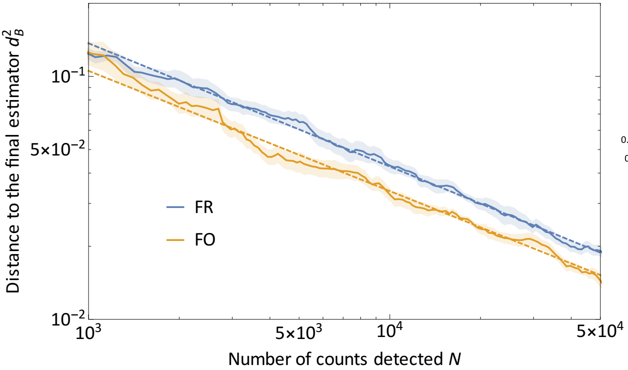


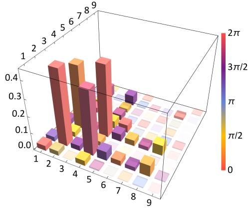
$\mathrm{Tr}\rho^2 = 0.94$

Advantage is $\approx$ 2.2 times.

FR $N^{-0.502}$ | FO $N^{-0.703}$

G.I.Struchalin et al. Phys. Rev. A **98**, 032330 (2018)





$\mathrm{Tr}\rho^2 = 0.74$

Advantage is ≈ 1.25 times.

FR $N^{-0.507}$ | FO $N^{-0.495}$

G.I.Struchalin et al. Phys. Rev. A **98**, 032330 (2018)

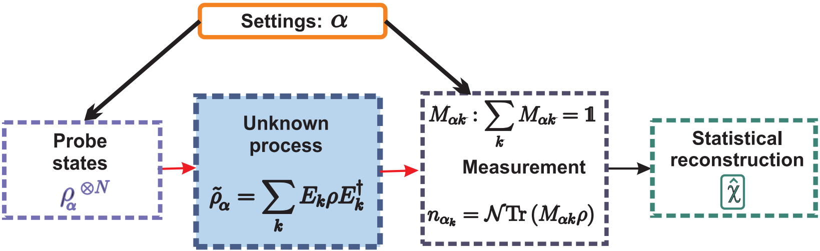
Outline

Quantum state engineering in SPDC with shaped pump

Quantum state tomography of spatial states

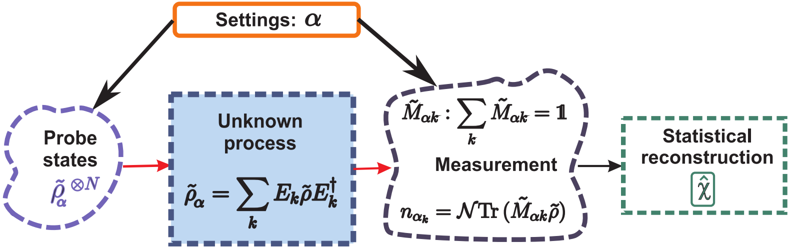
- Adaptive quantum tomography of high-dimensional states
- Fighting SPAM errors with neural networks
- Experimental shadow tomography

SPAM errors

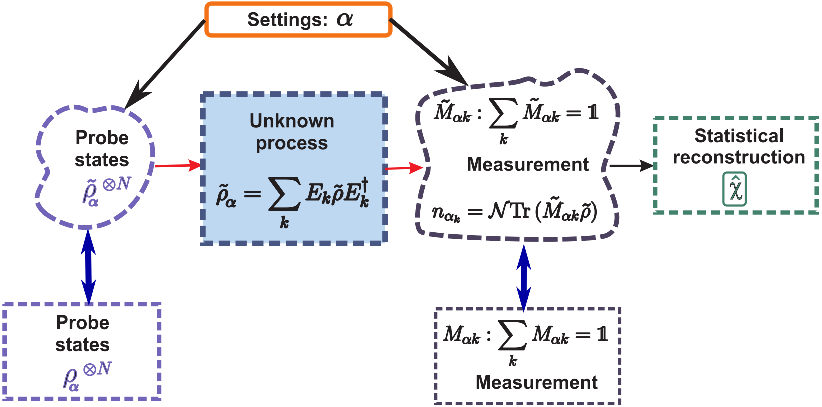


Settings: α
Probe states
$\rho_\alpha^{\otimes N}$
Unknown process
$\tilde{\rho}_\alpha = \sum_k E_k \rho E_k^\dagger$
$M_{\alpha k} : \sum_k M_{\alpha k} = \mathbb{1}$
Measurement
$n_{\alpha_k} = \mathcal{N} \mathrm{Tr} \left(M_{\alpha k} \rho \right)$
Statistical reconstruction
$\hat{\chi}$

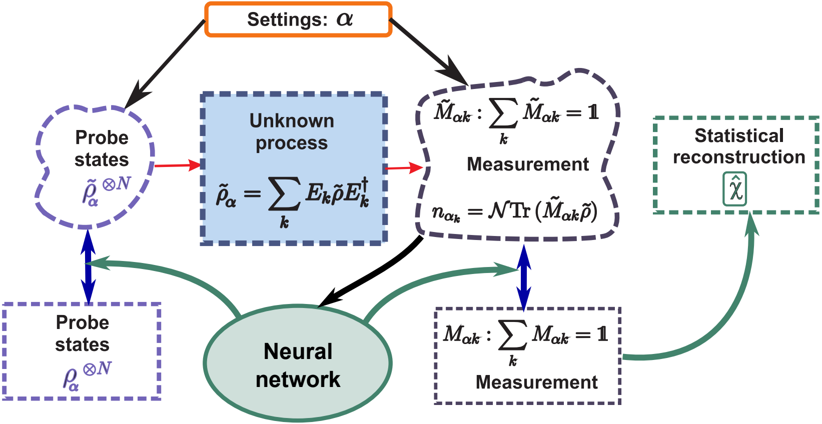
SPAM errors



Probe states and measurements are corrupted by errors

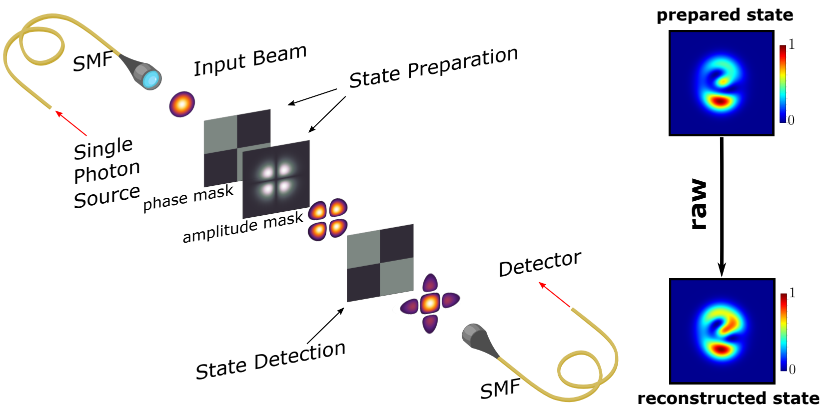


We need to recover «ideal» data to perform reconstruction



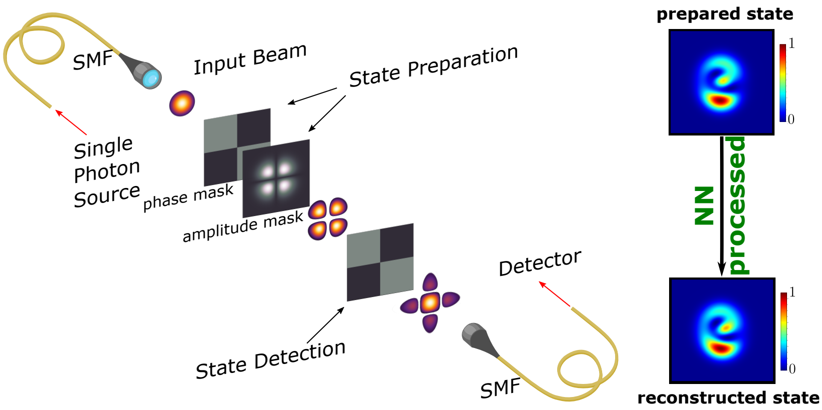
A feed-forward neural network is trained to perform denoising

Experiment with OAM of photons



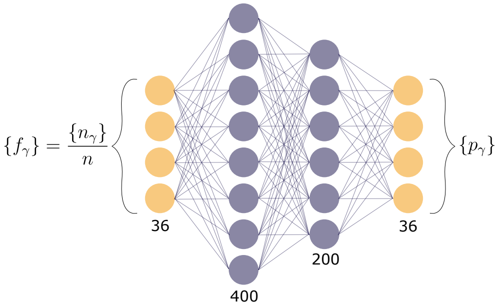
Reconstruction of a 6-dimensional spatial photonic state

Experiment with OAM of photons



Reconstruction of a 6-dimensional spatial photonic state

Neural network architecture



- Input data: *empirical frequencies* f_γ
- Output: *predicted probabilities* p_γ
- *Expected (ideal) probabilities* $\mathbb{P}_\gamma$

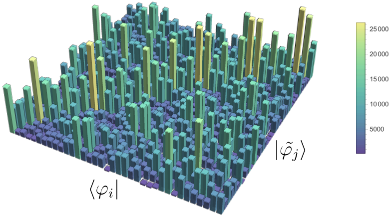
The NN is trained to minimize the **KL divergence**

$$L = \sum_{i=1}^{N} \sum_{\gamma=1}^{d^2} \mathbb{P}_\gamma^i \log\left(\frac{\mathbb{P}_\gamma^i}{p_\gamma^i}\right)$$

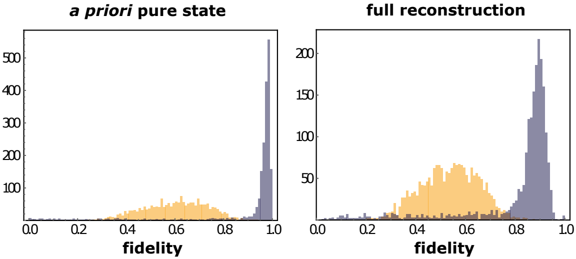
Reconstruction results: raw vs. processed

Cross-talk between the projectors leads to fidelity reduction

$$P_j^i = |\langle\varphi_i|\,\tilde{\varphi}_j\rangle|^2$$



Fidelity distribution for 1000 random states



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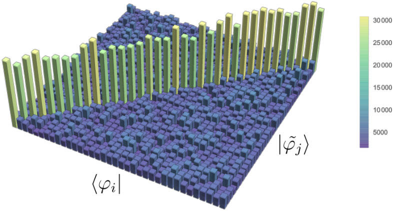
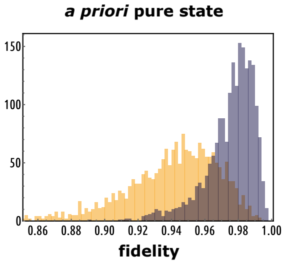
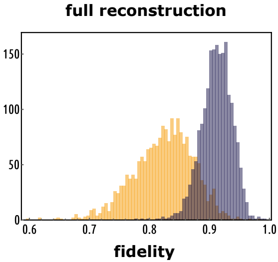
Reconstruction results: raw vs. processed

Even with pre-calibration by standard process tomography **the NN still performs better!**



Fidelity distribution for 1000 random states





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Outline

- Full state tomography of a d-dimensional quantum states requires $N \sim d^2$ measurements
- This is intractable in high dimensions

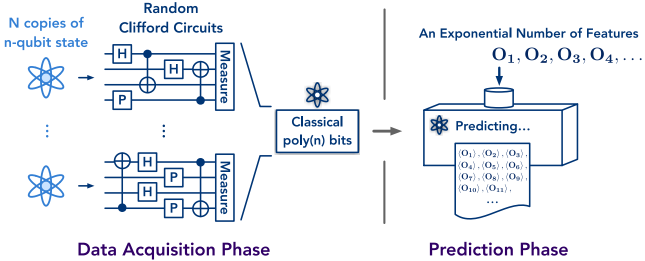
Is there any way around this problem?

Yes, if we do not need full information about the state!

What if you only want to estimate some expectation values?

$$\langle O_i \rangle = \mathrm{Tr}\, O_i \rho$$

It is possible to do with $N \sim \log d$ measurements



S. Aaronson, Proceedings of the 50th Annual ACM SIGACT STOC 2018, 325–338 (2018)
H.-Y. Huang, R. Kueng, arXiv:1908.08909v2 (2019)

Algorithm:

- "Classical shadow of ρ": $\{(|\psi_0\rangle\langle\psi_0|, f_0), \ldots, (|\psi_N\rangle\langle\psi_N|, f_N)\}$, where $f_i = N_i/N$ are the observed outcome frequencies
- Obtain the linear inversion estimate:

$$\hat{\rho} = (d+1)\sum_i f_i |\psi_i\rangle\langle\psi_i| - \mathbb{I}$$

- Compute the estimate $\hat{O}_i = \mathrm{Tr}\, O_i \hat{\rho}$

It works given appropriate symmetry in the choice of $|\psi_k\rangle$ (they should form a 3-design)

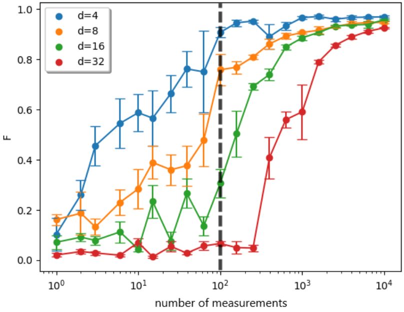
H.-Y. Huang, R. Kueng, arXiv:1908.08909v2 (2019)

Let us choose $O = |\psi\rangle\langle\psi|$

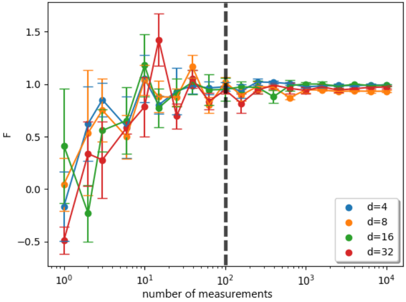
We will obtain an **estimate of fidelity** to a given state

Experimental results for spatial states:

MLE



Shadow



G.I.Struchalin et al. *In preparation*

Conclusions

- **Spatial modes of SPDC photons** are a useful tool for quantum experiments
- We have implemented several ideas to **engineer the spatial entanglement**
- One can perform **arbitrary projective measurements** and do full state reconstruction in **dimensions up to 36**
- **Feed-forward neural networks** may be used for pre-processing of data in quantum tomography to mitigate the effect of SPAM errors
- Full state tomography is redundant for many purposes and may be replaced with cleverer **resource-saving protocols**