ITMO UNIVERSITY

Quantum Metrology beyond Heisenberg limit with bright quantum solitons

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MSU Quantum Technology Centre Seminar

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Plan of the talk

УНИВЕРСИТЕТ ИТМО

- *Introduction: Quantum metrology tasks and state of the art ;*
- *Quantum metrology approach in optics and atom optics with Bose-Einstein condensates;*
- *Quantum metrology with condensate solitons;*
- *Quantum soliton Josephson Junction (SJJ) device as a new powerful tool for quantum metrological applications ;*
- *Lossy quantum metrology with solitons, Quantum Fisher information approach;*
- *Current progress in experiment for quantum metrology with solitons*
- *Summary*

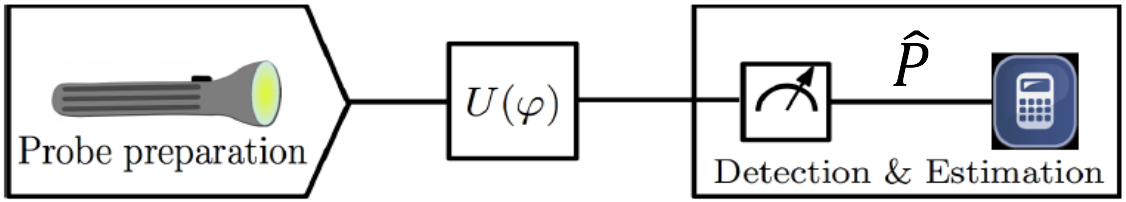
L. Mach. *zeitschr. f. Instrakde* 1892
L. Zehnder. *zeitschr. f. Instrakde* 1891

Quantum metrology: tasks and problems

УНИВЕРСИТЕТ ИТМО

The aim of quantum metrology is development of methods and devices to measure some physical parameter at the level of fundamental quantum noise limit, determined by some uncertainty relations.

A schematic diagram of φ parameter measurement and estimation



Propagation error is

$$\Delta\varphi = \frac{\sqrt{\left\langle \left(\Delta\hat{P}\right)^2 \right\rangle}}{\left|\frac{\partial < P >}{\partial\varphi}\right|}$$

The main questions:

1. *What is the transformation $U(\varphi)$ for measurement and estimation of φ which allows to obtain highest precision ?*
2. *How we can prepare quantum state of probe to achieve best measurement precision for φ?*
3. *What is the most effective scheme of detection the $\hat{P}$ observable?*

Some historical remarks

УНИВЕРСИТЕТ ИТМО

Early Studies – measurement of weak forces

- *V.B. Braginskii*, On the Limits Which Determine the Possibility of Measuring Gravitational Effects JETP (1963)
- V. B. Braginskii, Yu. I Vorontsov . Quantum-mechanical limitations in macroscopic experiments and modern experimental technique, *Sov. Phys. Usp.* (1975)
- W. H. Press , K. S. Thome , Gravitational-Wave Astronomy. Preprint, Caltech, 1972.
- C.M. Caves, C. M., K.S. Thorne, et al . *On the measurement of a weak classical force coupled to a quantum-mechanical oscillator. Rev. Mod. Phys. (1980)*
- D. Wineland, et al, Squeezed atomic states and projection noise in spectroscopy, Phys. Rev. A (1994)

Vladimir Braginsky

Current studies

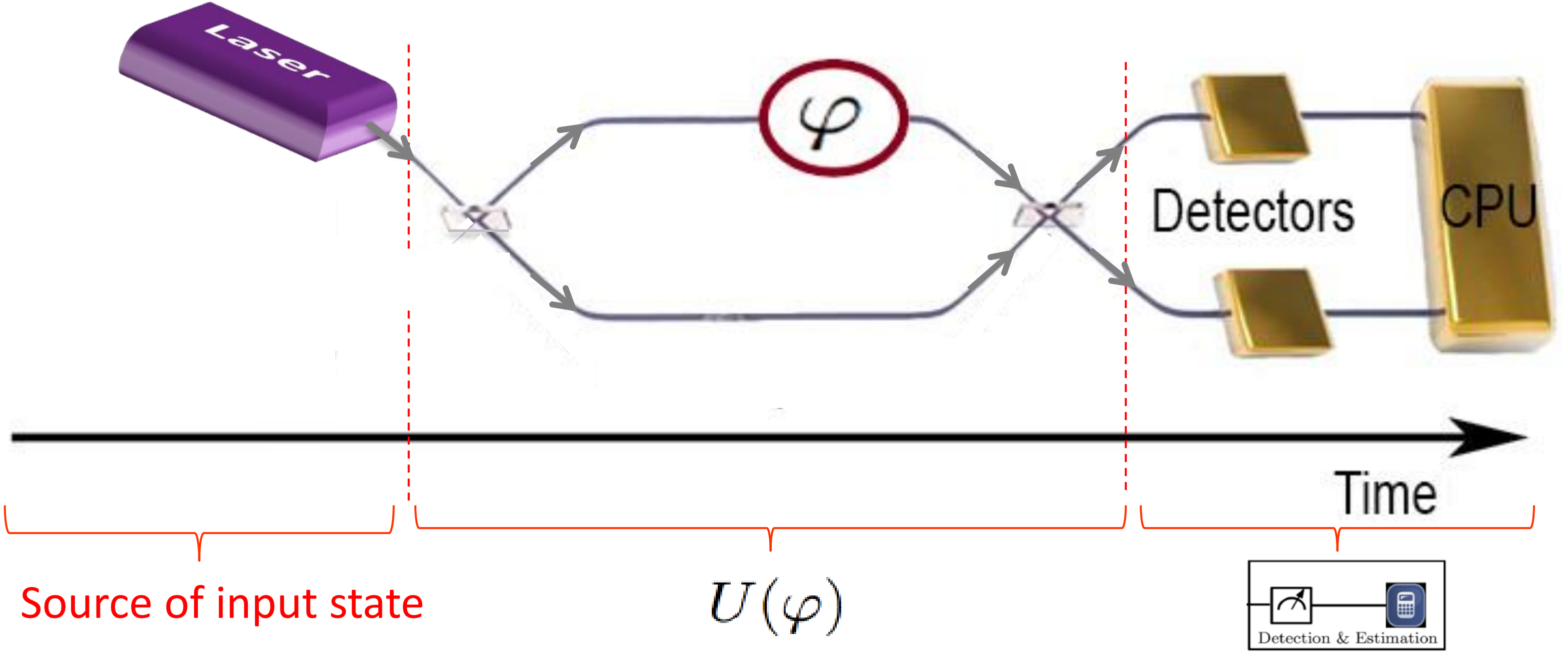
- *J. P. Dowling, K. P. Seshadreesan*, Quantum Optical Technologies for Metrology, Sensing, and Imaging," J. of Lightwave Technology, (2015)
- *L. Pezze, et al*, Quantum metrology with non-classical states of atomic ensembles , Rev. Mod. Phys. (2018)
- *C.L. Degen, F. Reinhard, and P. Cappellaro* , Quantum sensing , Rev. Mod. Phys. (2017)

Quantum Information approach

- *S. L. Braunstein, C. M. Caves, G.J. Milburn*, Annals of Physics (1996) – Quantum phase estimation
- *Giovannetti, Seth Lloyd, and Lorenzo Maccone*, PRL (2006) – Quantum metrology strategies
- *S. Boixo, Steven T. Flammia, Carlton M. Caves, JM Geremia*, PRL (2007) – Quantum Nonlinear metrology
- *U. Dorner, R. Demkowicz-Dobrzanski*, , et al, PRL (2009) - Optimal Quantum Phase Estimation (lossy QM)

A. Alodjants, S. Arakelian, Grav. and Cosmology (1999) – Quantum polarization state exploiting in interferometry

Sketch on Mach-Zehnder interferometer



To measure some physical variable one needs to measure precisely the phase-shift, $\varphi \ll 1$, in the arms of the interferometer

Interferometry with coherent probe $|\alpha>$

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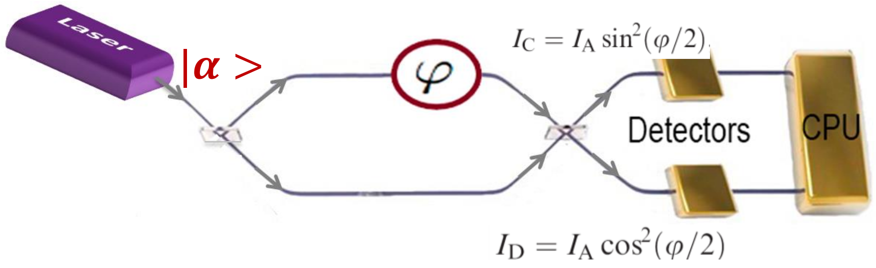
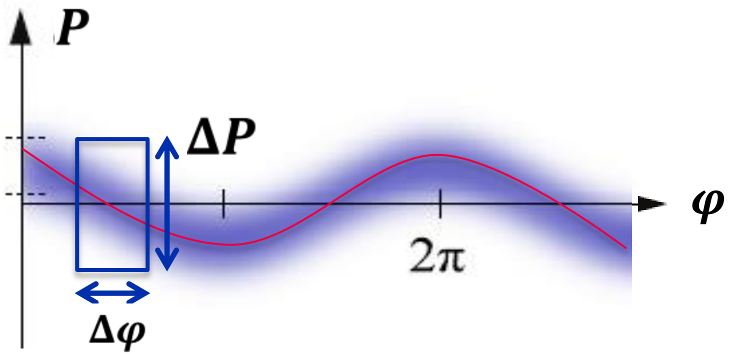
MZ-scheme



$N_C + N_D = N$ $\quad$ $P = N_C - N_D$



Quantum phase uncertainty with coherent state :

Uncertainty relation

$$\Delta\varphi \Delta P \geq 1$$

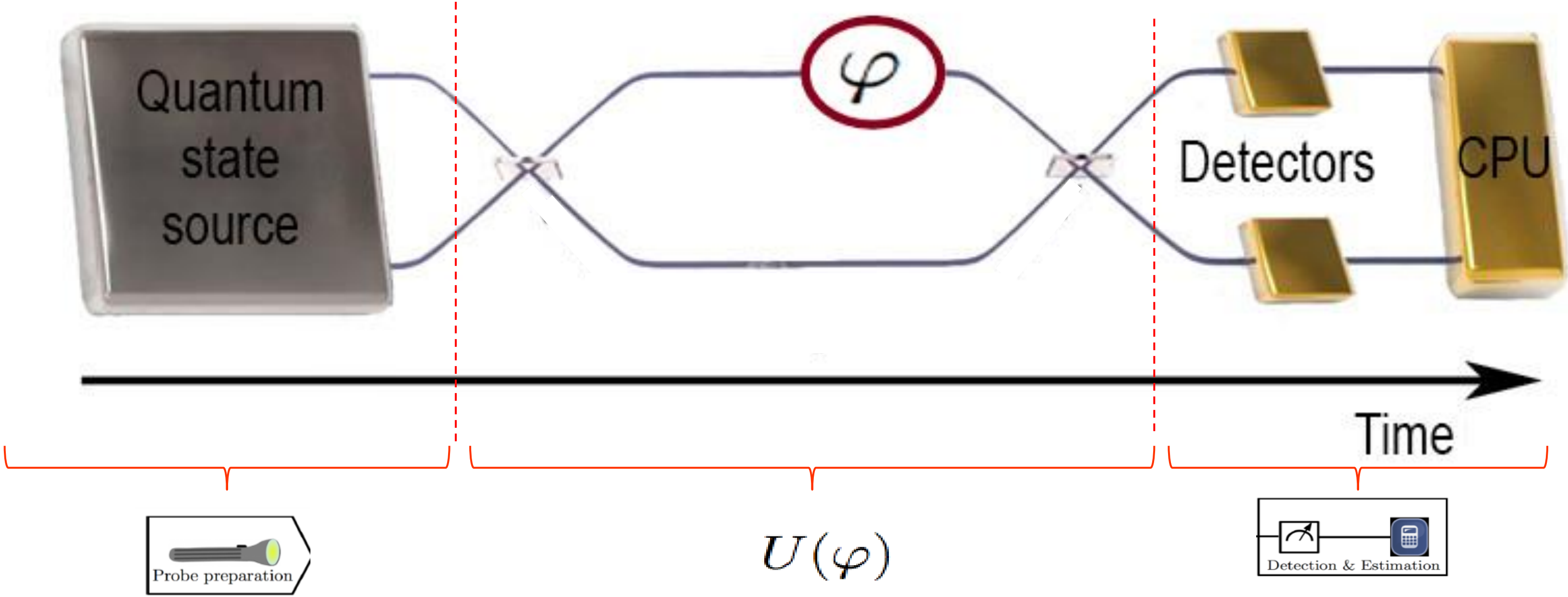
$$\Delta\varphi = \frac{\Delta P}{|\partial P/\partial\varphi|} = \frac{\Delta P}{I_A|sin(\varphi)|} = \frac{\Delta P}{I_A} \quad For\ \varphi = \frac{\pi}{2}$$

$$\Delta P = \sqrt{\left\langle \left(\Delta\hat{P}\right)^2 \right\rangle} = \sqrt{N}$$

$$\boxed{\Delta\varphi_{SQL} = \frac{1}{\sqrt{N}}}$$

Standard Quantum Limit for phase measurement

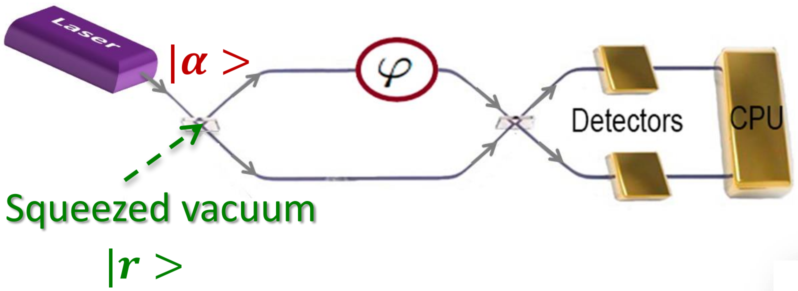
Metrology with non-classical states



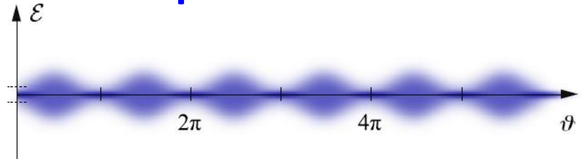
To measure some physical variable one needs to measure precisely the phase-shift, $\varphi \ll 1$, in the arms of the interferometer

Metrology with noise suppression (GEO600, KAGRA, LIGO, Virgo)

MZ- interferometer with squeezed input



Squeezed vacuum



Roman Schnabel, Physics Reports 684 (2017)

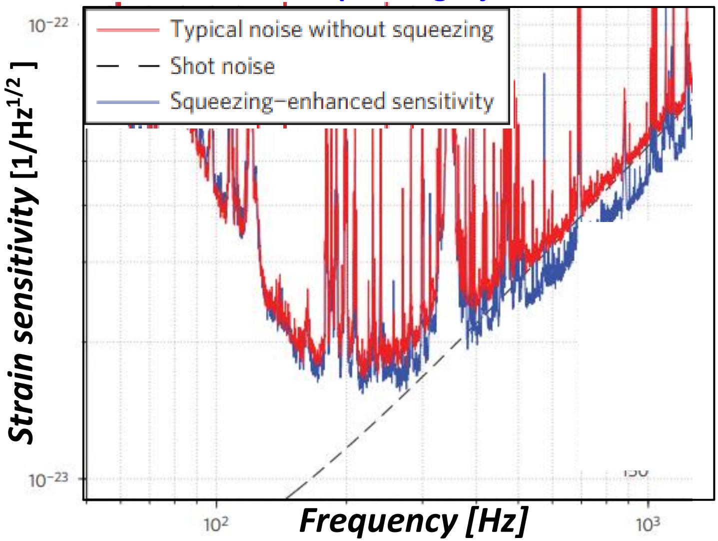
$$\Delta\varphi = \frac{1}{N^{3/4}}$$

$$N \equiv |\alpha|^2 \gg 1$$

$$r \gg 1$$

LIGO, Michelson interferometer

Strain sensitivity of the H1 detector measured with and without squeezing injection



Currently maximal squeezing in Lab is 15dB

Linear metrology

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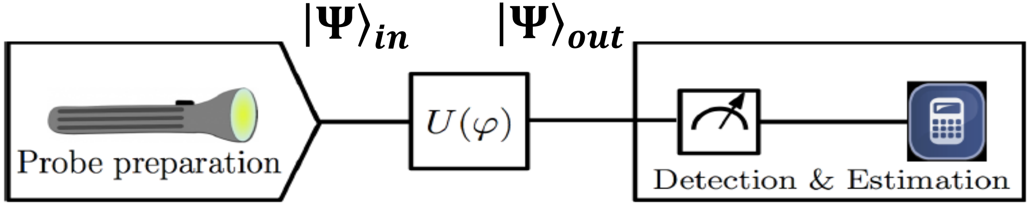

$$U(\varphi) = e^{i\varphi N}$$

N – number of particles

Heisenberg limit
with maximally path-entangled
NOON-state

$$|N\rangle_A|0\rangle_B + e^{i\varphi N}|0\rangle_A\,|N\rangle_B$$

$$\frac{1}{N} \leq \Delta\varphi \leq \frac{1}{\sqrt{N}}$$

Standard Quantum Limit (SQL),
with a two-mode
coherent input state $|\Psi\rangle$

Some recent experiments

- ✓ Heonoh Kim et al *Three-photon NOON states generated by photon subtraction from double photon pairs* Optics Express 17, 19720 (2009)
- ✓ I. Afek et al *High-NOON States by Mixing Quantum and Classical Light* Science **328**, 879 (2010)
- ✓ L. A. Rozema et al *Scalable Spatial Superresolution Using Entangled Photons* PRL **112**, 223602 (2014)
- ✓ S. T Merkel, F. Kwilhelm *Generation and detection of NOON states in superconducting circuits* NJP **12**, 093036 (2010)

Vital question:
Is it possible to obtain NOON-like "robust" state for mesoscopic $N \geq 50$ number of particles?

A problem: *NOON*-state with $N > 3$ are hard to create!

Nonlinear metrology: Super-Heisenberg limit

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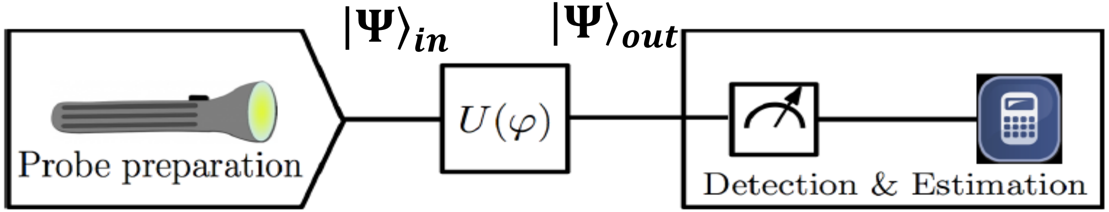


$$\boxed{U(\varphi) = e^{i\varphi N^m}}$$

φ – nonlinear phase-shift per particle

Super-Heisenberg limit with maximally path-entangled states

$$\boxed{\frac{1}{N^m} \le \Delta\varphi \le \frac{1}{N^{m-1/2}}}$$

Quantum limit with with two-mode coherent input state

For Kerr-like medium $m = 2$

$$|\Psi\rangle_{in} \Rightarrow \chi^{(3)} \Rightarrow |\Psi\rangle_{out}$$

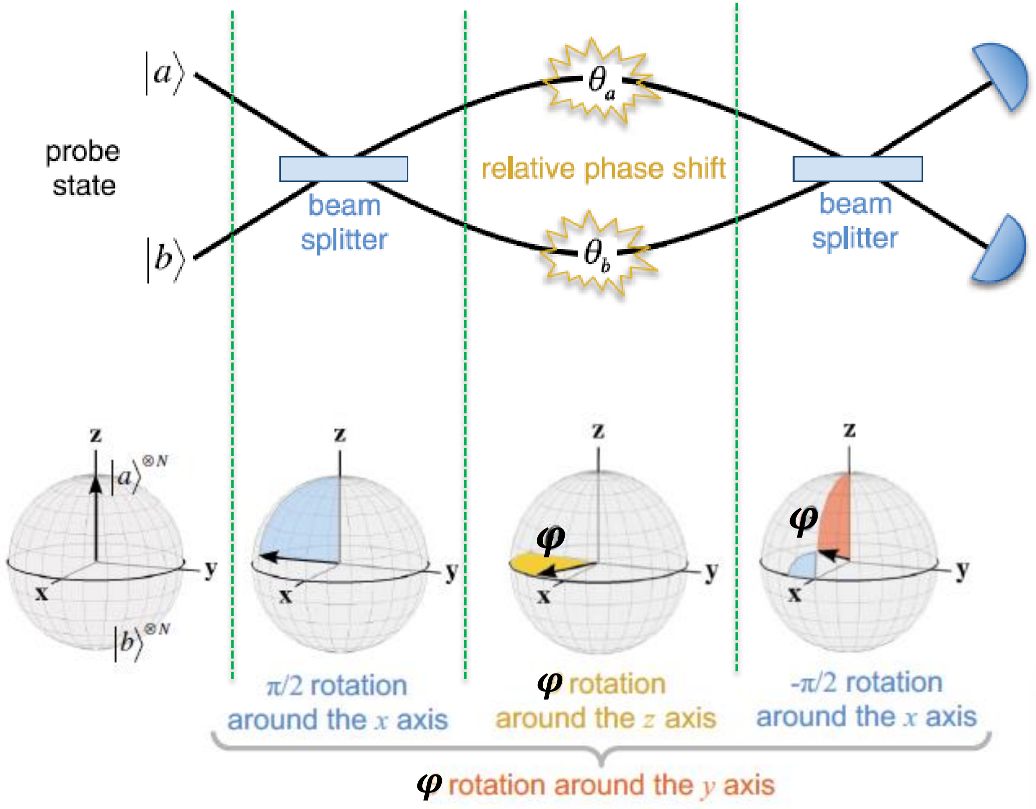
$$\frac{1}{N^2} \le \Delta\varphi \le \frac{1}{N^{3/2}}$$

Vital question:
Can we beat this limit in practice?

Some papers

- Sergio Boixo et al *Generalized Limits for Single-Parameter Quantum Estimation PRL **98**, 090401 (2007)*
- M. Napolitano et al *Interaction-based quantum metrology showing scaling beyond the Heisenberg limit Nature **471** , 486 (2011)*

Quantum metrology with atomic condensates



In a Mach-Zehnder interferometer two spatial modes $|a\rangle$ and $|b\rangle$ are combined on beam splitter, followed by a relative phase shift $\varphi = \theta_a - \theta_b$ between the two arms, and finally recombined on a second beam splitter.

Equivalent representation of MZ interferometer operations as rotations of the collective spin on the Bloch sphere. The initial state here $|a\rangle^{\otimes N}$ is pointing toward the north pole. The full sequence is equivalent to the rotation of an φ angle around the y axis.

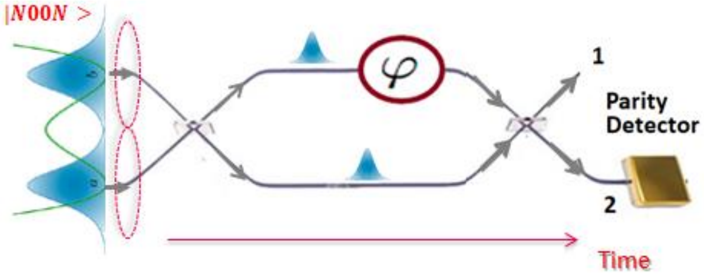
Condensates possess high nonlinearity in Tonks-Girardeau (1D) regime at mesoscopic number of particles

L. Pezze, et al, Rev. Mod. Phys. (2018)

Quantum Metrology with solitons

УНИВЕРСИТЕТ ИТМО

Linear metrology approach, φ is linear phase



$$\hat{P}_2 = e^{i\pi\hat{a}_2^\dagger\hat{a}_2}$$

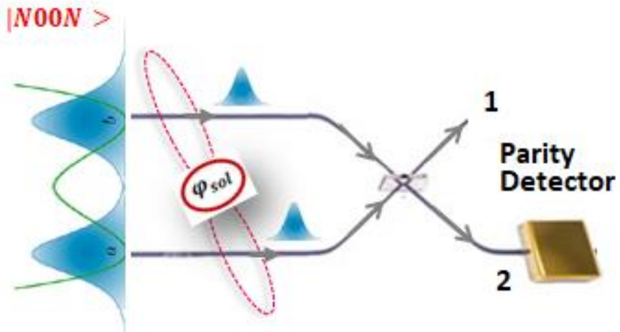
We can do a particle counting measurement and assign parity to be +1 if the number is even or-1 if the number is odd.

Ultimate Propagating error is

$$\Delta\varphi = \frac{1}{N}$$

Coupled solitons allow to create entangled Fock states approaching N00N -states

Non-linear metrology approach



Ultimate Propagating error is

$$\Delta\varphi = \frac{1}{N^3}$$

φ_{sol} Is nonlinear phase between solitons

G. S. Thekkadath , B. A. Bell, I. A. Walmsley, and A. I. Lvovsky, Quantum 4, 239 (2020)

Quantum approach to solitons

Quantum field theories for solitons

Elliott H. Lieb and Werner Liniger, *Exact Analysis of an Interacting Bose Gas. I, II* Physical Review 130: 1605–1624 (1963)

Faddeev, L. D., & Korepin, V. E.. *Quantum theory of solitons. Physics Reports, 42(1), 1–87* (1978)

Solitons in Quantum optics

P. D. Drummond and S. J. Carter, “Quantum-field theory of squeezing in solitons," J. Opt. Soc. Am. B **4**, 1565 (1987).

Y. Lai and H. A. Haus, Quantum theory of solitons in optical fibers. Phys. Rev. A **40**, 844 (1989)

A. V. Belinskii and A. S. Chirkin, Quantum theory of nonlinear propagation of Schrodinger solitons: squeezed states and sub-Poisson statistics Zh. Eksp. Teor. Fiz. 98,407418 (1990)

S. R. Friberg, et al, Observation of Optical Soliton Photon-Number Squeezing," Phys. Rev. Lett. **77**, 3775 (1996).

Solitons under matter light interaction

Ray-Kuang Lee and Yinchieh Lai, Quantum squeezing and correlation of self-induced transparency solitons, Phys. Rev. A **80**, 033839 (2009)

T. Y. Golubeva, Yu. M. Golubev, et al. Quantum Fluctuations in a Laser Soliton. *Opt. Spectrosc.* **128,** 505 (2020)

BEC solitons

C. J. Pethick and H. Smith, *Bose-Einstein Condensation in Dilute Gases,* (Cambridge University, 2008)

S. Raghavan and G.P. Agrawal *Switching an d self-trapping dynamics of Bose – Einstein Solitons* *J. Mod. Opt.* **47** 1155–69 (2000)

Atomic bright solitons

Schrodinger Equation

$$\mathrm{i}\frac{\partial}{\partial t}\psi = -\frac{1}{2}\frac{\partial^2}{\partial x^2}\psi - u\,|\psi|^2\psi$$

$u = 2\pi|a_{sc}|/a_0$ characterizes atomic nonlinearity

a_{sc} is an s-wave scattering length

a_0 is a characteristic trap size

Classical non-moving bright soliton

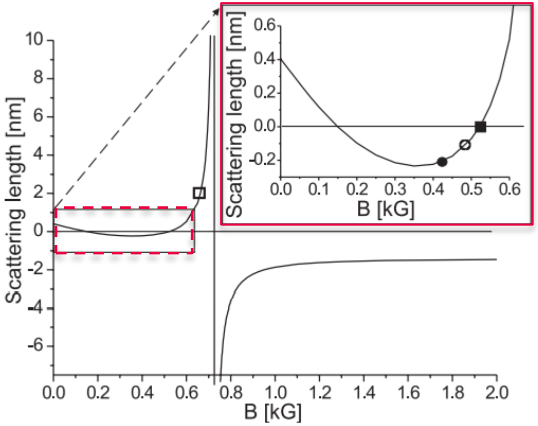
$$\psi(x,\, t) = \frac{N\sqrt{u}}{2}\,\mathrm{sech}\left[\frac{Nux}{2}\right]\mathrm{e}^{\mathrm{i}\frac{N^2u^2}{8}t}$$

$\psi(x,\, t)$ condensate 'wave-function

Normalization condition

$$\int|\psi|^2\mathrm{d}x = N$$

Feshbach resonance for $^7\mathrm{Li}$



Critical number of Li atoms when collapse occurs

$$uN_c \approx 4.2.$$
$$N_c|a_{sc}| \simeq 1.105\ \mu\mathrm{m}$$

$$5.2 \times 10^3$$

The number of particles is mesoscopic !

- ✓ Strecker K E, et al, *Nature* 417 150–3 (2002)
- ✓ Khaykovich L, et al , *Science* 296 1290 (2002)

Soliton quantization in single mode approximation

УНИВЕРСИТЕТ ИТМО

For BEC Gaussian state

Solution ansatz $\hat{\psi}(x,t) = \hat{a}(t)\Psi(x)$

$$\Psi = \frac{\nu^{1/4}\sqrt{N}}{\pi^{1/4}} e^{-\nu x^2/2} e^{i\theta}$$

Spatial condensate wave function

- ✓ A.S. Parkins, D.F. Walls, Physics Reports (1998)
- ✓ J. R. Anglin and A. Vardi, Phys. Rev. A (2001)

Hamiltonian is

$$\hat{H} = \int dx \hat{\psi}^\dagger(x,t)\left(-\frac{1}{2}\frac{\partial^2}{\partial x^2} - \frac{u}{2}\hat{\psi}^\dagger(x,t)\hat{\psi}(x,t) + U_{tr}(x)\right)\hat{\psi}(x,t)$$

$$\hat{H}_{\mathrm{eff}} = \hbar\Omega_L\hat{N} - \hbar\Omega_2\hat{N}^2$$

$$\begin{cases} \hbar\Omega_L = \int dx \Psi^*(x)\left(-\frac{1}{2}\frac{\partial^2}{\partial x^2} + U_{tr}(x)\right)\Psi(x), \\ \hbar\Omega_2 = \frac{u}{2}\int dx|\Psi(x)|^4. \end{cases}$$

$$\hat{N} = \hat{a}^\dagger\hat{a}$$

For BEC *soliton* state

Solution ansatz

$$\psi(x,t) = \frac{N\sqrt{u}}{2}\operatorname{sech}\left[\frac{Nux}{2}\right]e^{i\frac{N^2u^2}{8}t}$$

Classical Hamiltonian is

$$H = \int dx \psi^*(x,t)\left(-\frac{1}{2}\frac{\partial^2}{\partial x^2} - \frac{u}{2}|\psi(x,t)|^2\right)\psi(x,t)$$

$$H = -\frac{N^3u^2}{24}$$

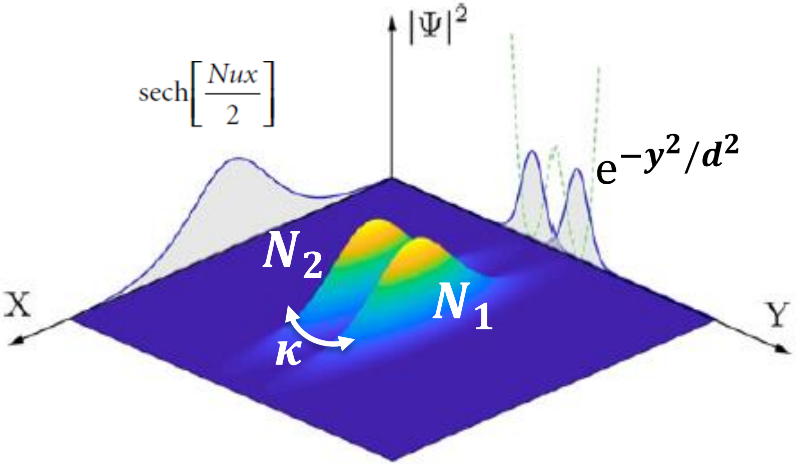
$$\hat{H}_{\mathrm{eff}} = -\hbar\Omega_3\hat{N}^3,$$

$$\hbar\Omega_3 = \frac{u^2}{24}$$

This approach is valid for characteristic trap size $a \gg |a_{sc}|\,N$

Soliton Josephson Junctions (SJJ)

УНИВЕРСИТЕТ ИТМО



$\boldsymbol{\kappa}$ is coupling rate

$z = (N_2 - N_1)/N$ is population imbalance

Semi-classical model

$$H_{\mathrm{SJJ}} \simeq -\frac{\Lambda_{\mathrm{eff}}}{2} z^2 - \sqrt{1-z^2}\ \cos[\theta]$$

$$\Lambda_{\mathrm{eff}} = \frac{u^2 N^2}{16\kappa_{\mathrm{eff}}}$$

$\kappa_{\mathrm{eff}} \equiv \kappa(1-0.21z^2)\sqrt{1-z^2}$

is effective tunneling parameter

$$\Lambda \equiv \frac{u^2 N^2}{16\kappa}$$

Is vital parameter

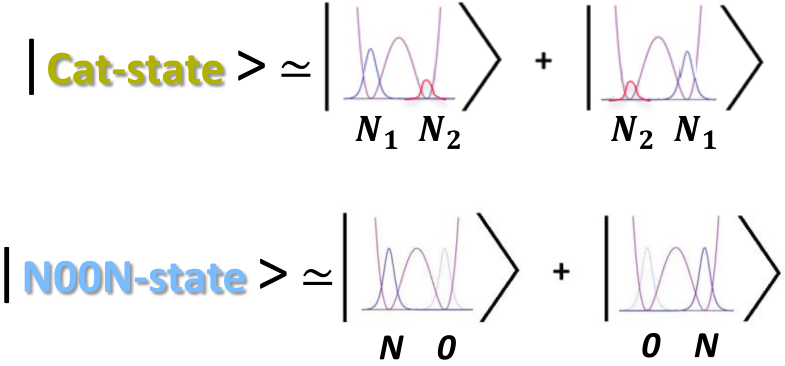
Self-tuning effect

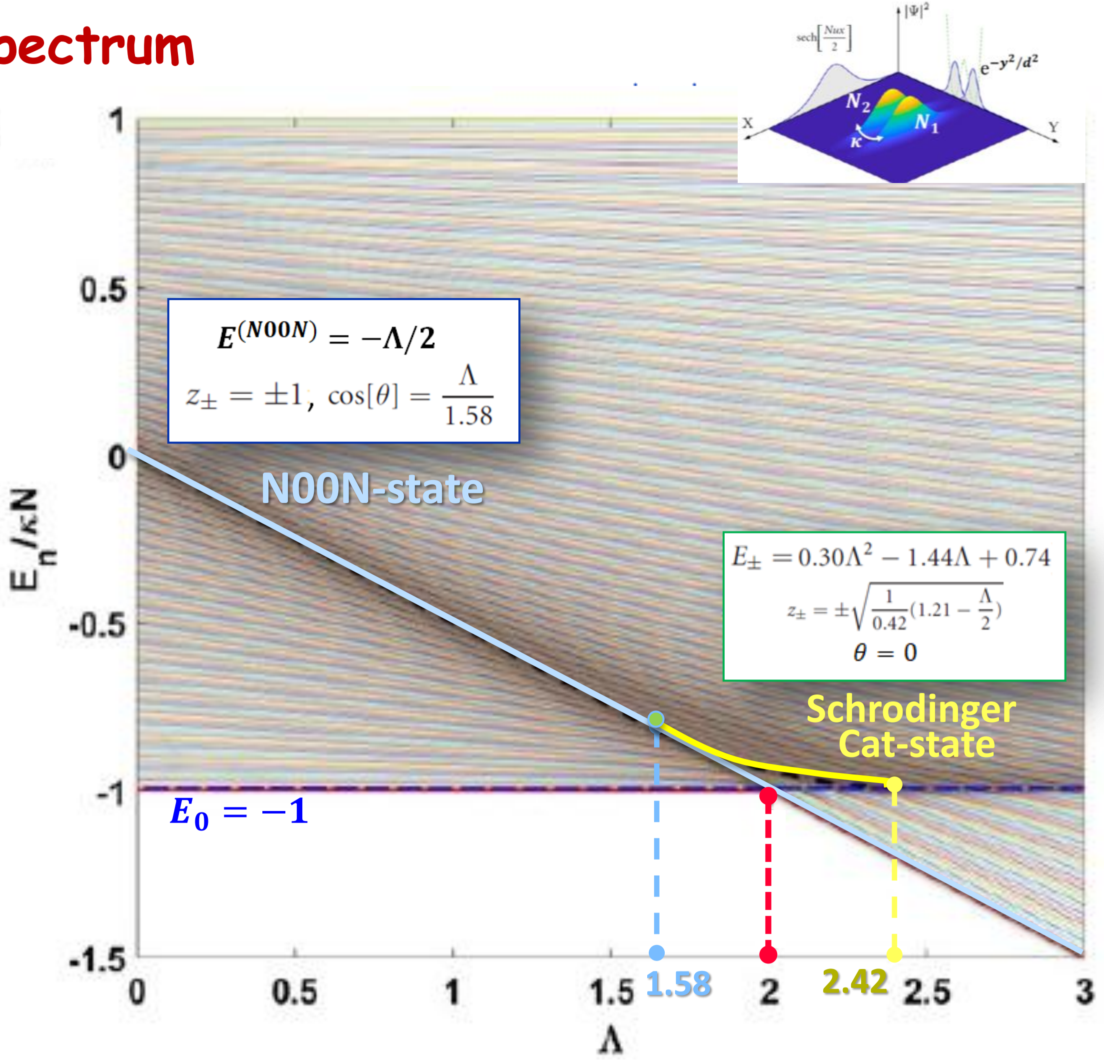
For $z^2 \ll 1$ we have $\boldsymbol{\kappa_{eff} \approx \kappa}$ and $\boldsymbol{\Lambda_{eff} \approx \Lambda}$ - similar to Gaussian BECs

For $z^2 \to 1$ we have $\boldsymbol{\kappa_{eff} \to 0}$ and $\boldsymbol{\Lambda_{eff} \to \infty}$ - ***the tunneling is effectively blocking!***

Quantum SJJ model; Energy spectrum

Dependencies of the quantum Hamiltonian eigen-energies, $E/\kappa N$, vs Λ-parameter for N = 300 particles





D V Tsarev, A P Alodjants, T.V. Ngo and Ray-Kuang Lee, New J. of Physics. V. 22. P. 113016 (2020)

Quantum SJJ model

УНИВЕРСИТЕТ ИТМО

Ground state in the Fock basis:

$$|\Psi\rangle = \sum_{n=0}^{N} A_n \, |\mathrm{n}\rangle_1 |\mathrm{N}-\mathrm{n}\rangle_2$$

Where $\sum_{n=0}^{N} |A_n|^2 = 1$

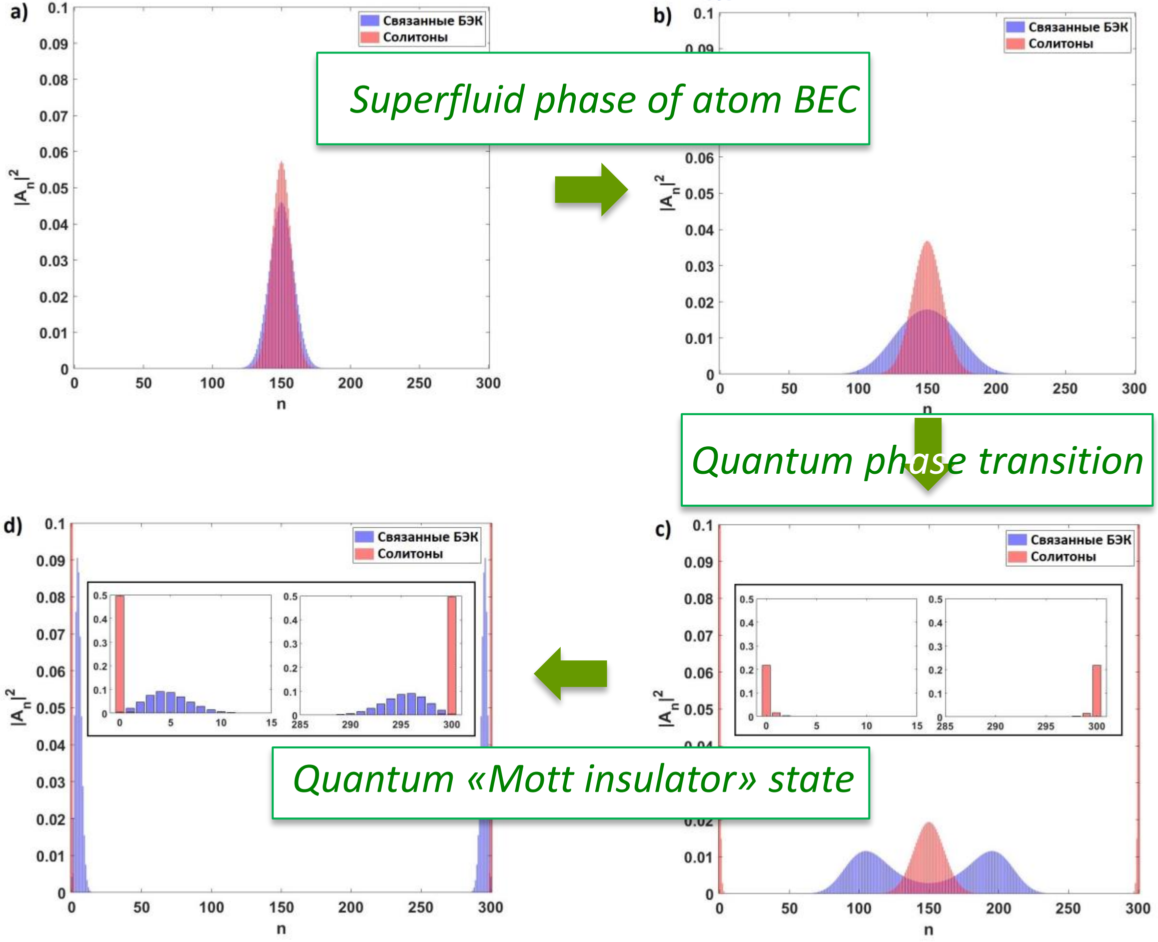
Coefficients A_n may be find from solution of Schrodinger equation

a) $\Lambda, \lambda \approx 0;$

b) $\Lambda = 2, \lambda = 1;$

c) $\Lambda \approx 2.009925, \lambda = 2.1;$

d) $\Lambda, \lambda = 4$



Haigh T J, Ferris A J and Olsen M K *Opt. Commun.* 283 3540–7 (2010)

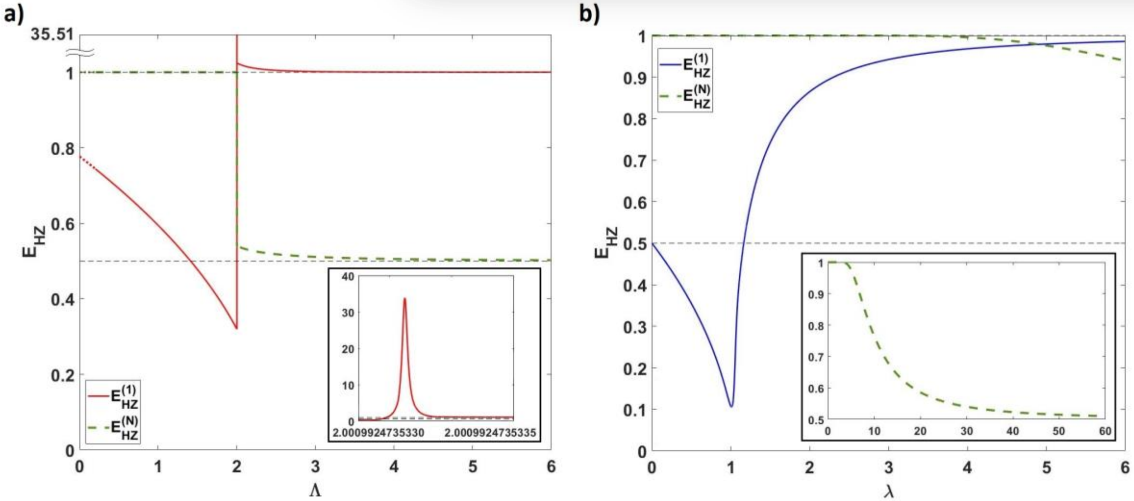
Multi-particle entanglement

Hillery-Zubairy m-th order entanglement criteria : $\mathbf{0 \leq E_{HZ}^{(m)} < 1}$, where

$$E_{\mathrm{HZ}}^{(m)} = 1 + \frac{\left\langle \hat{a}^{\dagger m}\hat{a}^{m}\hat{b}^{\dagger m}\hat{b}^{m}\right\rangle - \left|\left\langle \hat{a}^{m}\hat{b}^{\dagger m}\right\rangle\right|^{2}}{\left\langle \hat{a}^{\dagger m}\hat{a}^{m}\left(\hat{b}^{m}\hat{b}^{\dagger m} - \hat{b}^{\dagger m}\hat{b}^{m}\right)\right\rangle}.$$

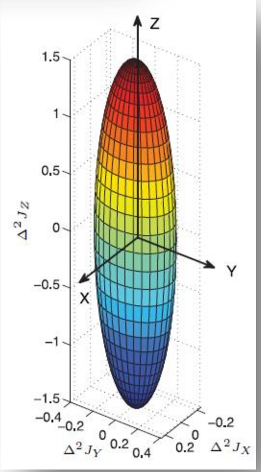
$$E_{\mathrm{HZ}}^{(1)} = \frac{\delta J_X + \delta J_Y}{\langle \hat{J} \rangle},$$

$$\delta J_j \equiv \left\langle \left(\Delta \hat{J}_j\right)^2 \right\rangle = \left\langle \hat{J}_j^2 \right\rangle - \left\langle \hat{J}_j \right\rangle^2$$



$E_{HZ}^{(1)} < 1$ characterizes quantum entanglement (planar spin-squeezing), m=1

$E_{HZ}^{(N)} = 0.5$ characterizes $N00N$-state formation, m=N

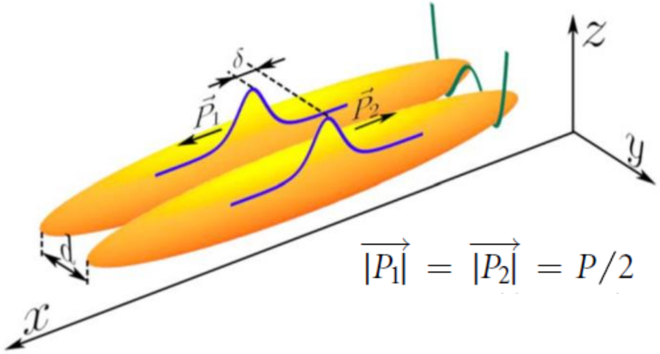


✓ He Q Y, Drummond P D, Olsen M K and Reid M D 2012 *Phys. Rev.* A 86 023626
✓ H. M. Wiseman, S. J. Jones, and A. C. Doherty Phys. Rev. Lett. 98, 140402 – Published 6 April 2007

Nonlinear Metrology with solitons; Results

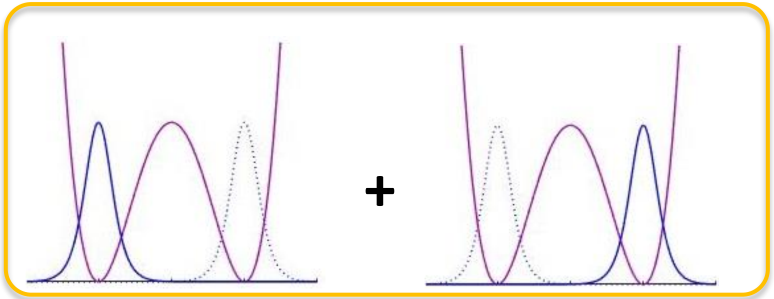
УНИВЕРСИТЕТ ИТМО

Scheme for counterpropagating solitons



P – *relative momentum of solitons;*

δ – *relative coordinate of solitons.*



Superposition *N00N*-state:

$$|N00N\rangle = \frac{1}{\sqrt{2}}\left(|N0\rangle + e^{-i\varphi_{sol}}|0N\rangle\right),$$

Relative phase

$$\varphi_{sol} = \arccos(-0.625\Lambda)$$

D V Tsarev , T.V. Ngo, Ray-Kuang Lee, and AP Alodjants, *Nonlinear quantum metrology with moving matter-wave solitons* , New J. Phys. 21, 083041, (2019)

Propagation error for soliton parameters

Soliton phase $\varphi_{sol} \approx \frac{\pi}{2}N + 0.63N^3\Theta$

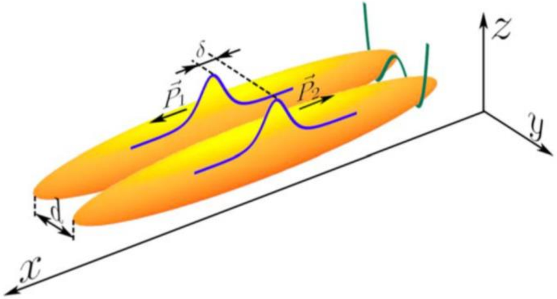
where $\Theta = u^2/16\kappa$ is parameter that we can estimate

$$\Delta\Theta \propto \frac{1}{N^3}$$

Phase difference of two interfering matter waves $\varphi = P\delta/\hbar$

Standard Quantum Limit for displacement measurement with $\Delta\varphi_{SQL} = 1/\sqrt{N}$

$$\Delta\delta_{SQL} = \frac{\hbar}{\sqrt{NP}}$$

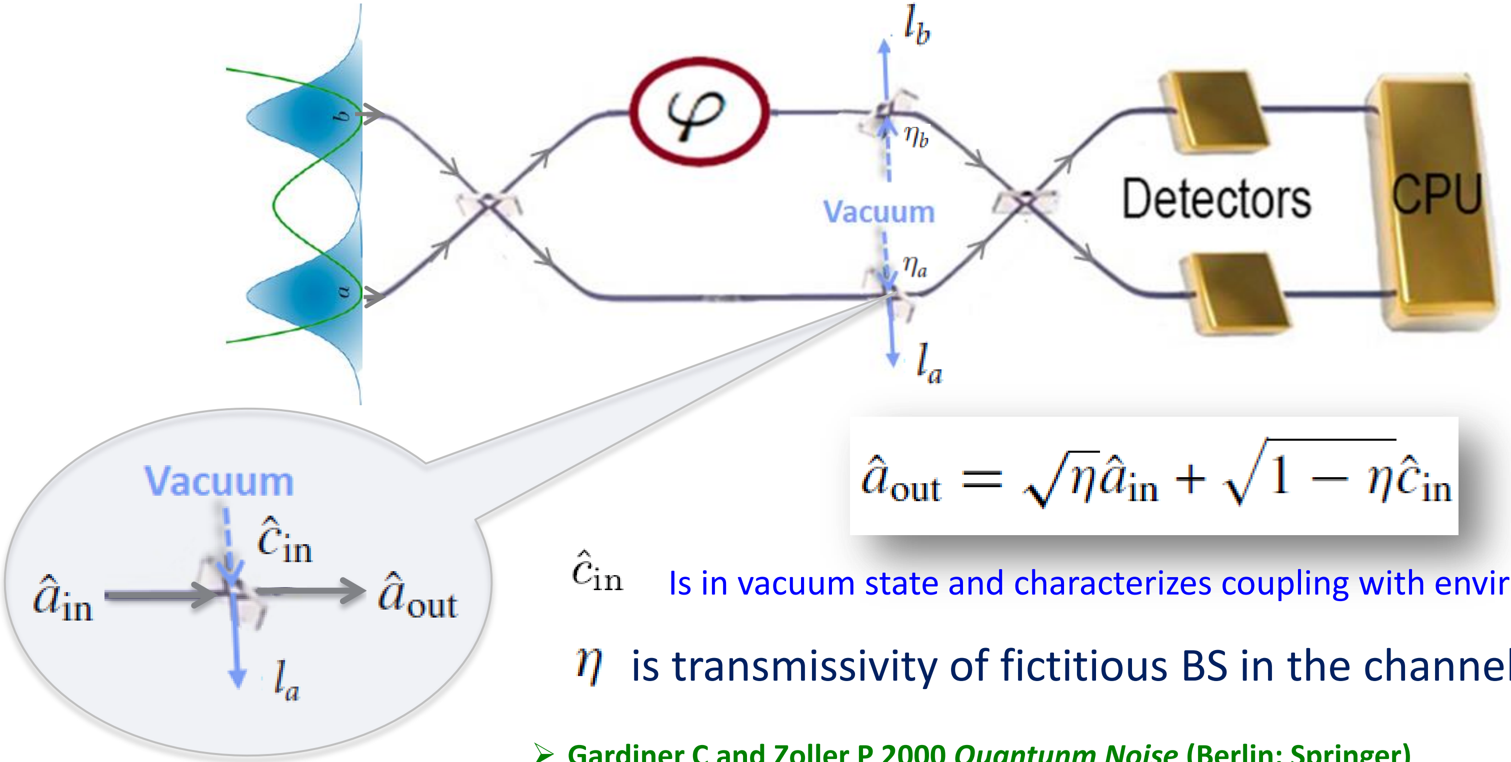


Propagation error for solitons displacement

$$\Delta\delta \propto \frac{\Theta}{N^3 P}$$

Lossy Quantum Metrology

Fictitious Beam Splitters method



$$\hat{a}_{\text{out}} = \sqrt{\eta}\hat{a}_{\text{in}} + \sqrt{1-\eta}\hat{c}_{\text{in}} \qquad 0 \leqslant \eta \leqslant 1$$

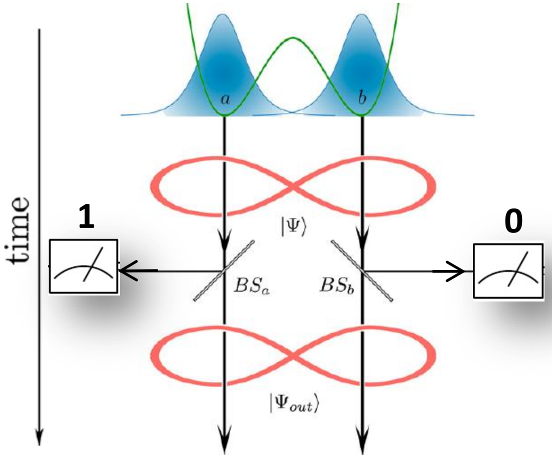
$\hat{c}_{\text{in}}$ Is in vacuum state and characterizes coupling with environment

η is transmissivity of fictitious BS in the channel

- Gardiner C and Zoller P 2000 *Quantunm Noise* (Berlin: Springer)
- U. Dorner, et al, PRL 102, 040403 (2009)
- A A Semenov *et al J. Phys. B: At. Mol. Opt. Phys.* 39 905 (2006)

Soliton Quantum State verification

Scheme of conditional state preparation



$\eta_a = \eta_b = 0.999$ $\Lambda \approx 2.000\,9925$

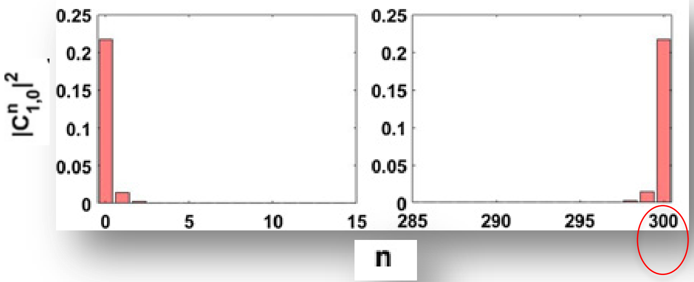
$N = 300$ $C^n_{l_a,l_b} \equiv \frac{1}{\sqrt{p}} A_n \sqrt{B^n_{l_a,l_b}}$

Conditional state

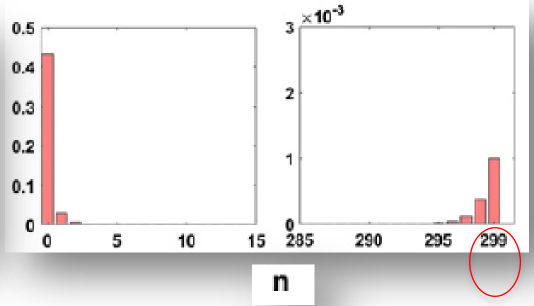
$$|\Psi_{\text{out}}\rangle = \frac{1}{\sqrt{p}} \sum_{l_b=0}^{N} \sum_{l_a=0}^{N-l_b} \sum_{n=l_b}^{N-l_a} A_n \sqrt{B^n_{l_a,l_b}} |N-n-l_a\rangle_a |n-l_b\rangle_b \, |l_a\rangle \, |l_b\rangle \, ,$$

$$B^n_{l_a,l_b} = \binom{N-n}{l_a} \binom{n}{l_b} \eta_a^{N-n} (\eta_a^{-1}-1)^{l_a} \eta_b^n (\eta_b^{-1}-1)^{l_b}.$$

$p = \sum_{l_b=0}^{N} \sum_{l_a=0}^{N-l_b} \sum_{n=l_b}^{N-l_a} |A_n|^2 B^n_{l_a,l_b}$ - normalization constant



Before measurement



After measurement

Population $\sqrt{N}$ times lower!

Fock State superposition

$$\left|\Psi^{(1,0)}_{\text{out}}\right\rangle = \frac{1}{\sqrt{p}} \sum_{n=0}^{N-1} A_n \sqrt{N-n} \sqrt{\eta^{N-1}(1-\eta)} |N-n-1\rangle_a |n\rangle_b$$

Quantum State verification

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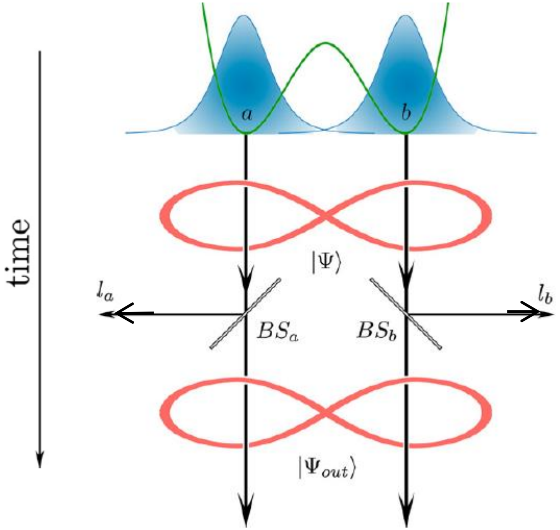
Scheme of solitons with losses

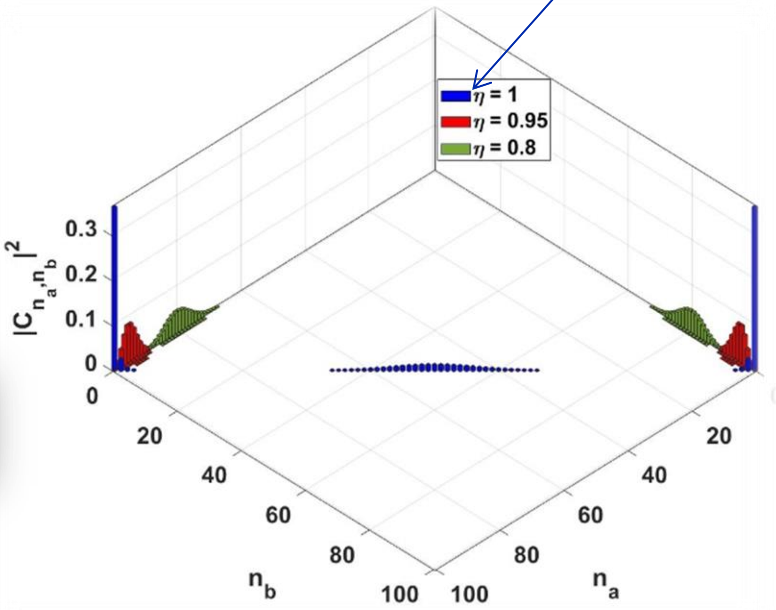


$$\Lambda = 2.003007086$$

The State

$$|\Psi_{\text{out}}\rangle = \sum_{l_b=0}^{N} \sum_{l_a=0}^{N-l_b} \sum_{n=l_b}^{N-l_a} C^n_{l_a,l_b} |N-n-l_a\rangle_a |n-l_b\rangle_b$$

Without losses



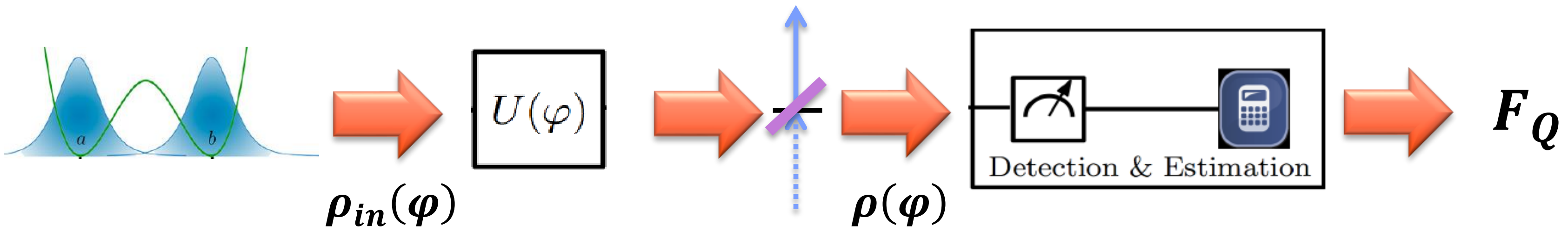
With losses

$$|\Psi_{out}\rangle = \sum_{n=0}^{N} \sqrt{\alpha_n} \left[|n\rangle_a |0\rangle_b + |0\rangle_a |n\rangle_b \right]$$

$$\alpha_n^{Gauss} = \frac{1}{2\sqrt{2\pi N(1-\eta)}} e^{-\frac{(n-N\eta)^2}{2N(1-\eta)}}$$

Quantum Fisher information approach

Scheme of metrology with losses



$$\hat{U}(\varphi) = e^{i\varphi(\hat{b}^{+}\hat{b})^{m}}$$

$m = 1$ — *Linear metrology approach*

$m = 3$ — *Non-linear metrology approach with solitons*

Quantum Cramer-Rao (QCR) bound; Fisher information

$$\Delta\varphi \geq \frac{1}{\sqrt{\nu}\sqrt{F_Q}}$$

where $F_Q = 4\left(\langle\psi'|\psi'\rangle - |\langle\psi'|\psi\rangle|^2\right) =< \Delta\left(\left(\hat{b}^{+}\hat{b}\right)^{m}\right)^2 >$

$|\psi'\rangle = \partial|\psi\rangle/\partial\varphi$ $\qquad$ ν Is a number of experimental runs

Fisher information Zoo in the presence of losses

Results (upper bound for FI)

$$\tilde{F}_Q = 4\left(\sum_{n=0}^{N} n^{2m}A_n^2 - \sum_{l_b=0}^{N}\sum_{l_a=0}^{N-l_b}\frac{\left(\sum_{n=l_b}^{N-l_a} n^m {C_{l_a,l_b}^{n}}^2\right)^2}{\sum_{n=l_b}^{N-l_a} {C_{l_a,l_b}^{n}}^2}\right)$$

$$\delta\phi_{min} = \frac{1}{\sqrt{F_Q}}$$

Standard interferometric limit (SIL) - $= 1/\sqrt{\eta N}$

Heisenberg Limit (HL) - $1/N$

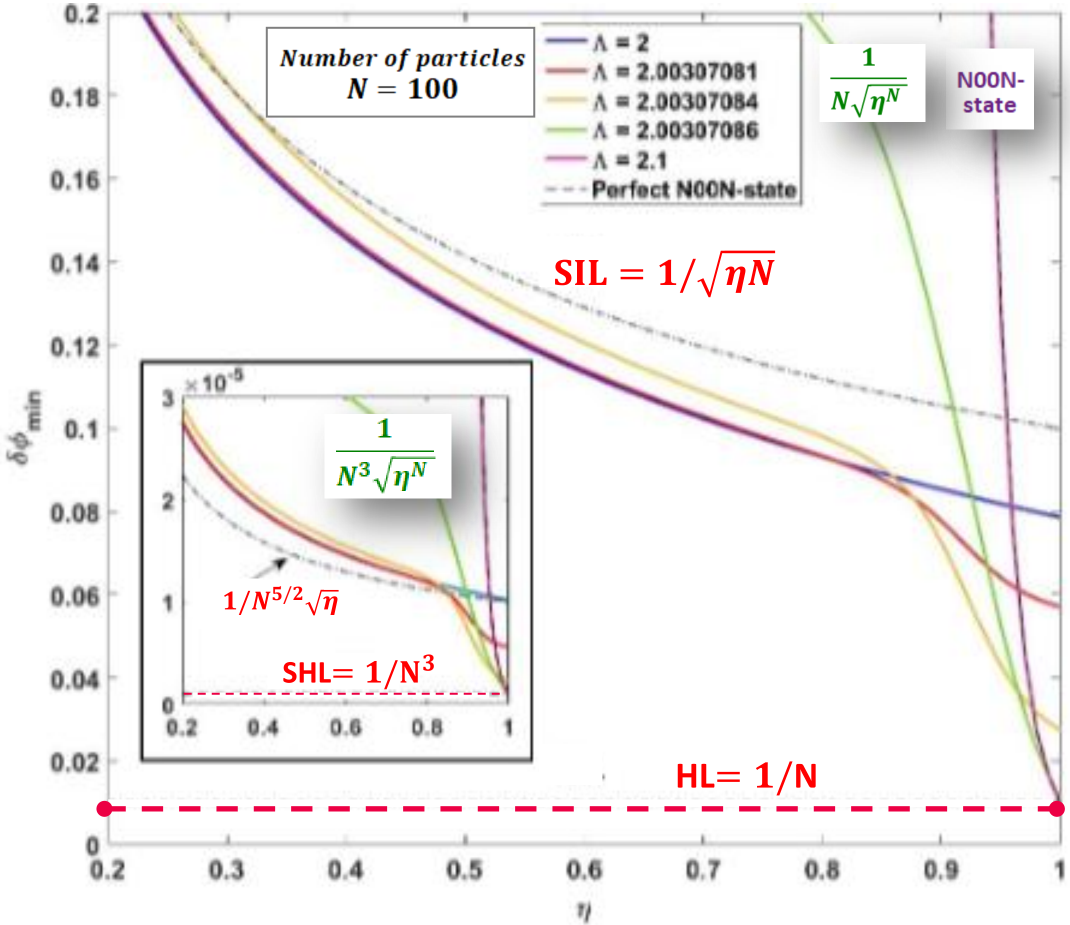
Non-linear interferometric limit (NIL) - $1/N^{5/2}\sqrt{\eta}$

Super-Heisenberg Limit (SHL) - $1/\mathrm{N}^3$

NOON state may be useful if

$\eta \geq e^{-2m/N}$, for m=1 , and m=3

For $\eta = 0.9$ $N_1 = 19$ *and* $N_3 = 57$



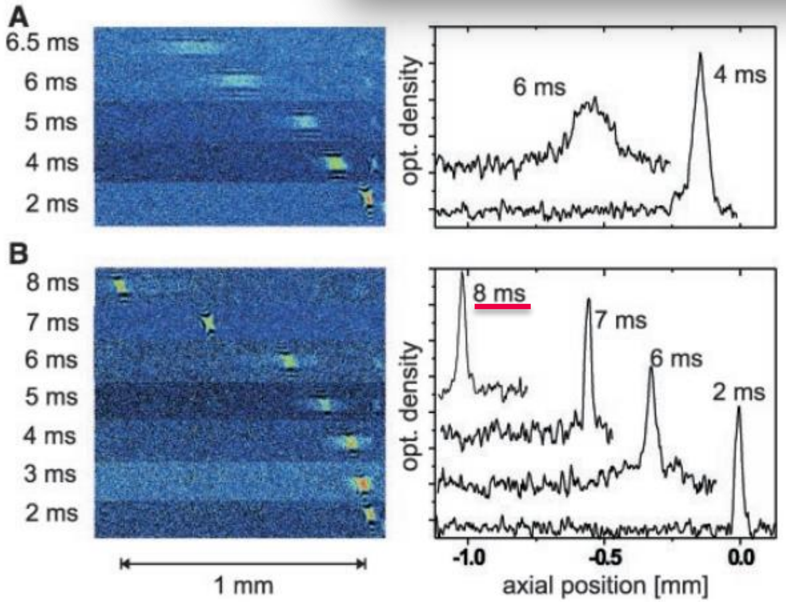
U. Dorner, et al, PRL 102, 040403 (2009)

Losses in condensate solitons in semi-classical limit

УНИВЕРСИТЕТ ИТМО

- One-body losses (exponential decay) $\gamma_1 = 0.2\ \mathrm{s}^{-1}$
- Three-body losses (non-exponential decay) $\gamma_3 = 5.2 \times 10^{-2}\ \mathrm{s}^{-1}$

Condition $\gamma_{1,3}\, t \ll 1$ is fulfilled in current experiments!

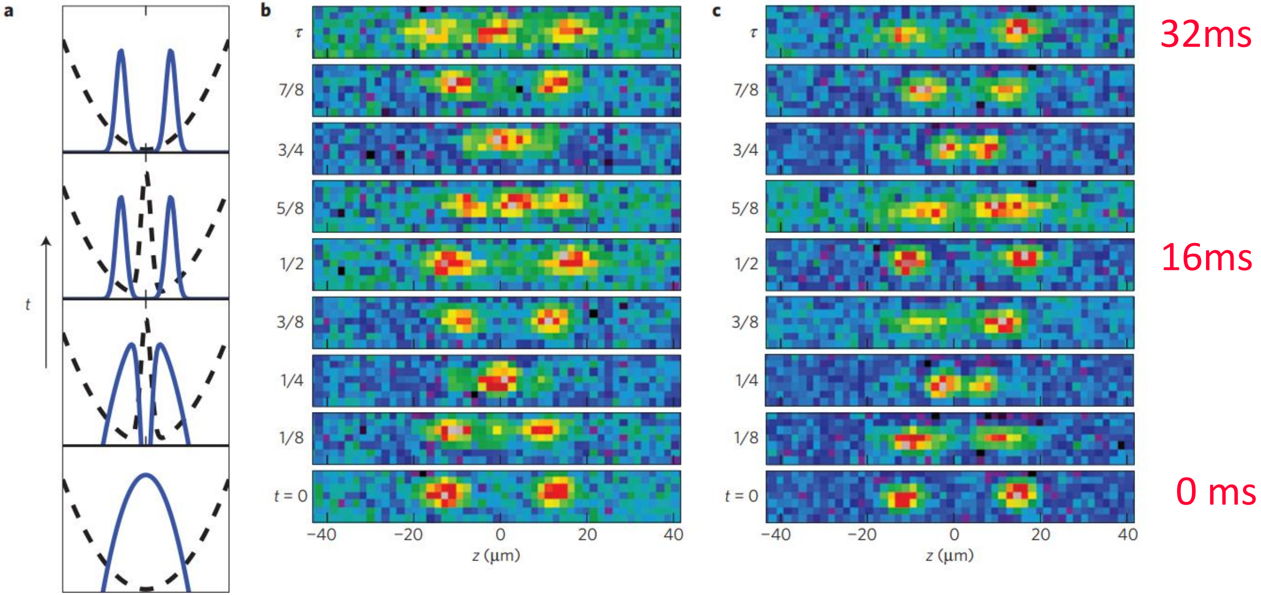


Absorption images at variable delays after switching off the vertical trapping beam. Propagation of an ideal BEC gas **(A)** and of a soliton **(B)** in the horizontal 1D waveguide in the presence of an expulsive potential. Propagation without dispersion over 1.1 mm is a clear signature of a soliton. Corresponding axial profiles are integrated over the vertical direction.

Lev Khaykovich, et al , *Science* 296 1290 (2002)

Atomic solitons collision

Schematic of the experiment and images of phase-dependent collisions

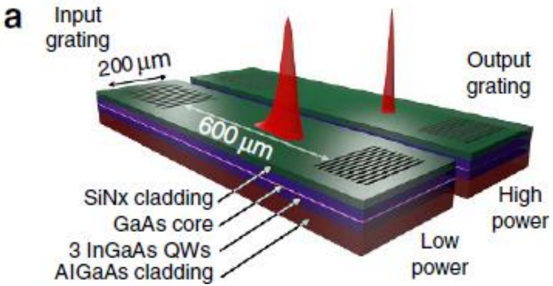


a) Couple of solitons formation via potential barrier. **b-c**) Time evolution of a soliton pair after the barrier is turned off. Solitons are accelerated towards the centre of the trap and collide at a quarter-period ($\tau = 2\pi/\omega_z$ =32ms). The density peak appearing at the centre-of-mass indicates that this is an in-phase ($\theta = 0$) collision. c, Similar to b, except the density node appearing at the centre-of-mass indicates an out-of-phase ($\theta = \pi$) collision.

Nguyen J, Dyke P, Malomed B A, et al Collisions of matter-wave solitons Nature Phys. 10, 10918-922 (2014)

What is about other solitons?

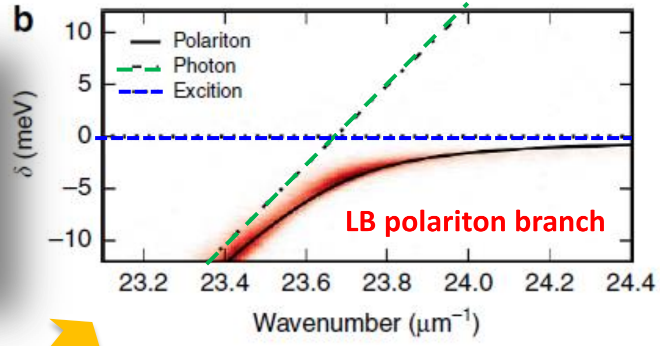
Exciton polariton solitons in waveguides (*P.M. Walker, et al, Nat.com.6, 2015*)

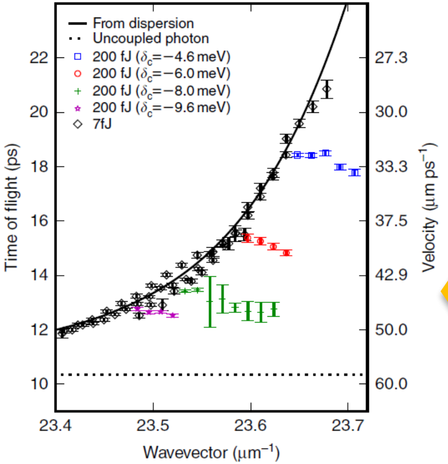


Strong coupling condition!

$$\gamma \ll \Omega_R$$

$$\gamma = 44.3\mu eV, \quad \Omega_R = 4.5 meV$$





(a) Schematic diagram of waveguide (the same waveguide is shown with pulses under low-power and high-power excitation conditions). **(b)** Angle-resolved photoluminescence spectrum showing emission from the LB polariton branch in red**.**

Measured time of flight of different wavevector components of pulse at low power (black diamonds) and high power (coloured symbols), and time of flight extracted from curvature of polariton dispersion (solid curve).

Refractive index is $n_2 = -1.6\times10^{-14}\,m^2W^{-1}$ It is three orders of magnitude larger than in planar AlGaAs waveguides in the weak coupling regime.

Conclusion

- We propose to use quantum bright solitons with mesoscopic number of particles for quantum metrological applications;
- We have shown ***for the first time*** that solitons allow SH phase estimation up to $\mathbf{1/N^3}$ level in the framework of a nonlinear metrology approach.
- We propose ***new*** soliton Josephson junction (SJJ) device that allows to create robust entangled Fock state which can be modeled by the N00N state for the losses less than $\mathbf{1-e^{-2m/N}}$, m=1 , and m=3.
- We have shown feasibility of quantum metrological applications of quantum solitons in current experiments with atomic solitons containing moderate (mesoscopic) number of particles.

L. Mach, *zeitschr. f. Instrakde* 1892
L. Zehnder, *zeitschr. f. Instrakde* 1891

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Recent publications

- ✓ **D V Tsarev, A P Alodjants, T.V. Ngo and Ray-Kuang Lee,** *Mesoscopic quantum superposition states of weakly-coupled matter-wave solitons*, **New J. of Physics**. V. 22. P. 113016 (2020)
- ✓ **D V Tsarev , T.V. Ngo, Ray-Kuang Lee, and AP Alodjants**, *Nonlinear quantum metrology with moving matter-wave solitons* , **New J. Phys**. 21, 083041, (2019)
- ✓ **D V Tsarev , S. M. Arakelian, You-Lin Chuang , Ray-Kuang Lee, and AP Alodjants** , *Quantum metrology beyond Heisenberg limit with entangled matter wave solitons*, **Optics Express**, 26, 19583 (2018)
- ✓ **A. Bazhenov, D. Tsarev, A. Alodjants,** *Temperature quantum sensor on superradiant phase transition*, **Physica B**, 579, 411879 (2020)

Thank you for attention!

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