QTB PQC LAB

Quantum Tomography Benchmarking

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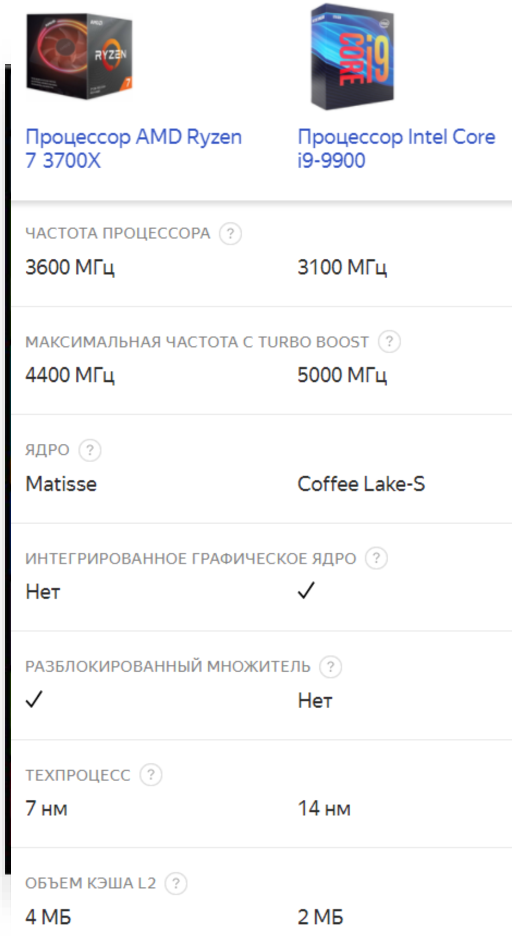
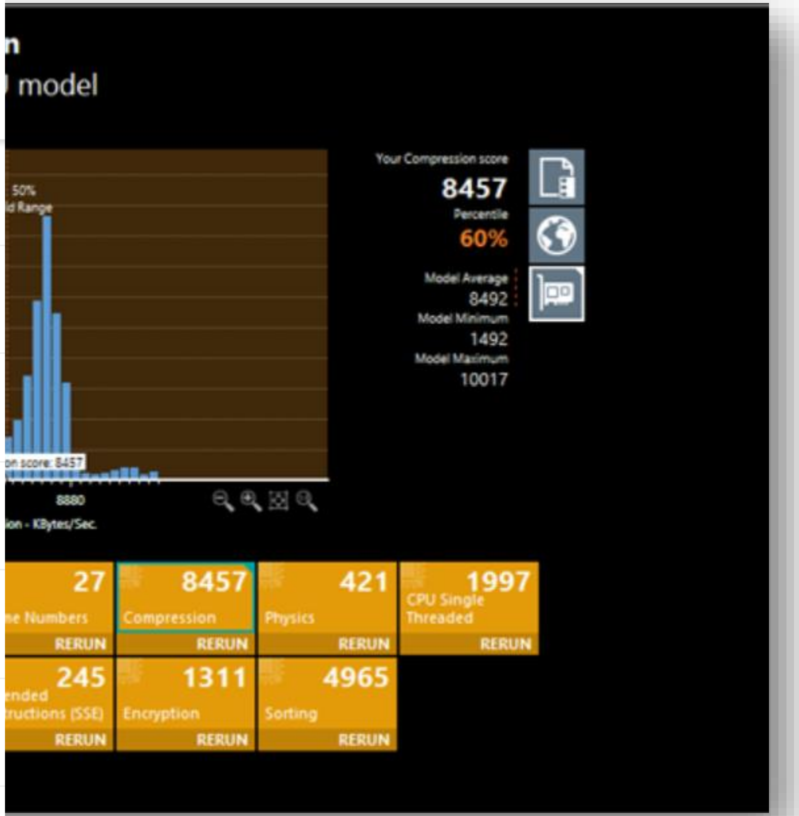
Quantum Tomography Benchmarking

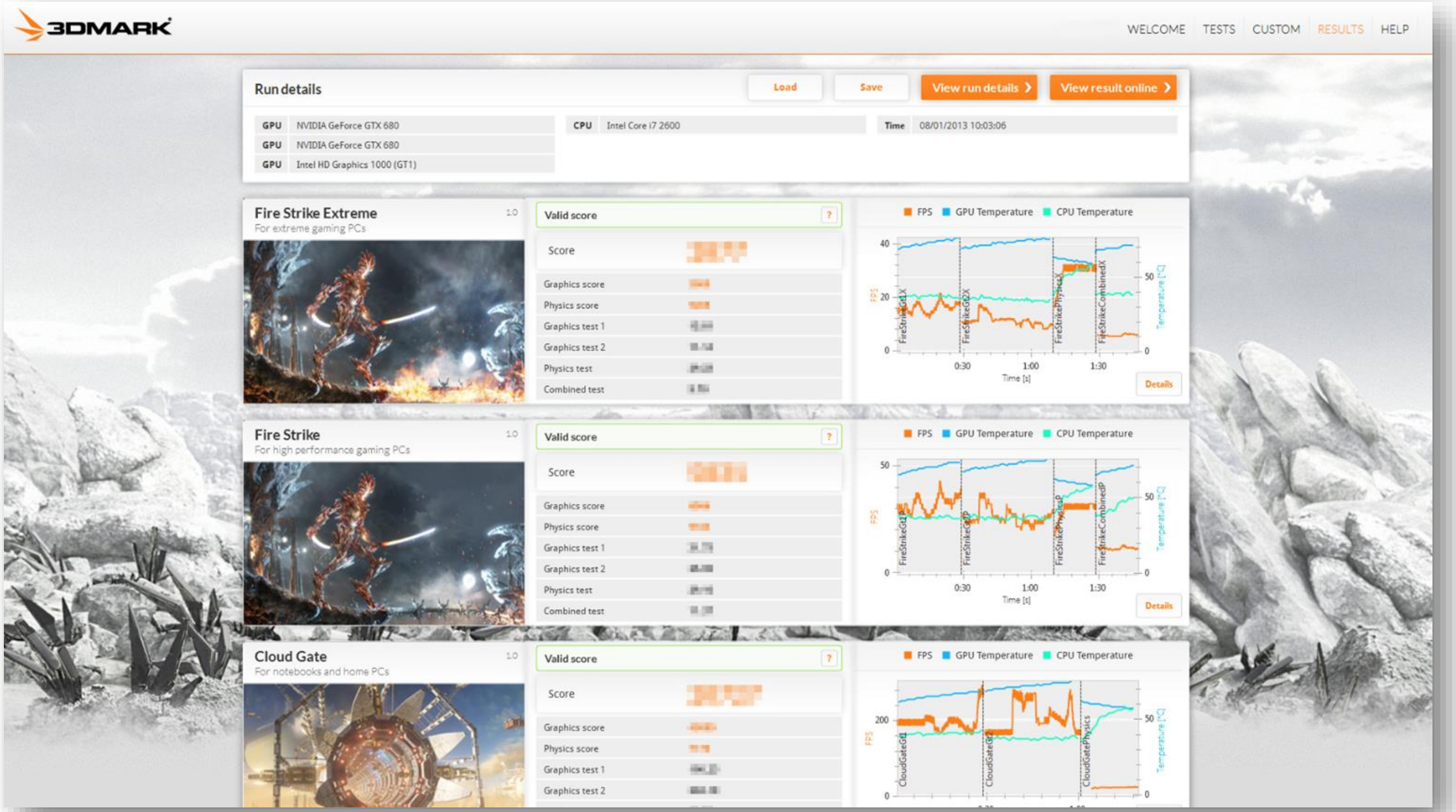
ARE YOU CHOOSING A NEW CPU?



Процессор AMD Ryzen 7 3700X	Процессор Intel Core i9-9900
ЧАСТОТА ПРОЦЕССОРА	
3600 МГц	3100 МГц
МАКСИМАЛЬНАЯ ЧАСТОТА С TURBO BOOST	
4400 МГц	5000 МГц
ЯДРО	
Matisse	Coffee Lake-S
ИНТЕГРИРОВАННОЕ ГРАФИЧЕСКОЕ ЯДРО	
Нет	✓
РАЗБЛОКИРОВАННЫЙ МНОЖИТЕЛЬ	
✓	Нет
ТЕХПРОЦЕСС	
7 нм	14 нм
ОБЪЕМ КЭША L2	
4 МБ	2 МБ



WHAT ABOUT GPU?

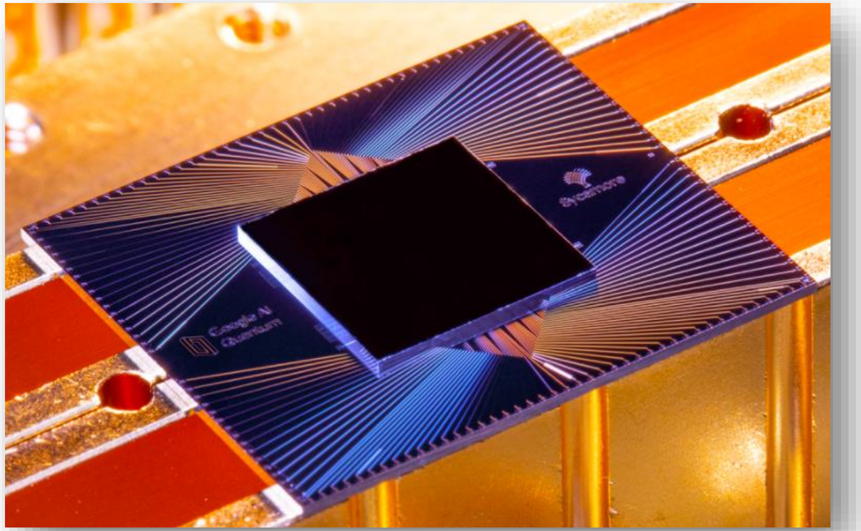


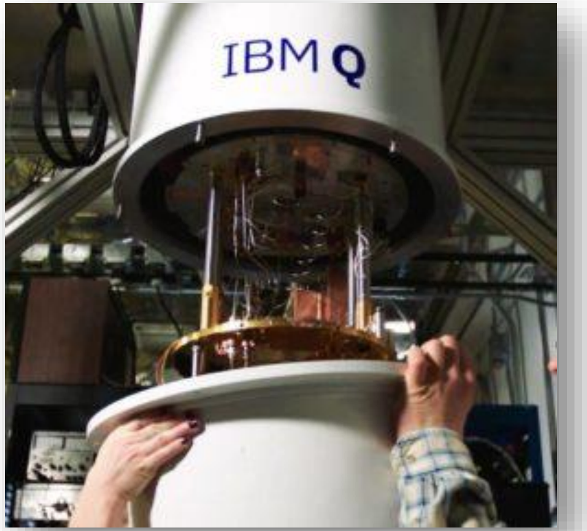
SUPERCOMPUTERS?

LINPACK benchmarks

Rank (previous)	Rmax Rpeak (PFLOPS)	Name	Model	CPU cores	Accelerator (e.g. GPU) cores	Interconnect	Manufacturer	Site country	Year	Operating system
1	442.010 537.212	Fugaku	Supercomputer Fugaku	158,976 × 48 A64FX @2.2 GHz	0	Tofu interconnect D	Fujitsu	RIKEN Center for Computational Science Japan	2020	Linux (RHEL)
2▼ (1)	148.600 200.795	Summit	IBM Power System AC922	9,216 × 22 POWER9 @3.07 GHz	27,648 × 80 Tesla V100	InfiniBand EDR	IBM	Oak Ridge National Laboratory United States	2018	Linux (RHEL)
3▼ (2)	94.640 125.712	Sierra	IBM Power System S922LC	8,640 × 22 POWER9 @3.1 GHz	17,280 × 80 Tesla V100	InfiniBand EDR	IBM	Lawrence Livermore National Laboratory United States	2018	Linux (RHEL)
4▼ (3)	93.015 125.436	Sunway TaihuLight	Sunway MPP	40,960 × 260 SW26010 @1.45 GHz	0	Sunway[26]	NRCPC	National Supercomputing Center in Wuxi China[26]	2016	Linux (Raise)
5▲ (7)	63.460 79.215	Selene	Nvidia	1,120 × 64 Epyc 7742 @2.25 GHz	4,480 × 108 Ampere A100	Mellanox HDR Infiniband	Nvidia	Nvidia United States	2020	Linux (Ubuntu)
6▼ (5)	61.445 100.679	Tianhe-2A	TH-IVB-FEP	35,584 × 12 Xeon E5–2692 v2 @2.2 GHz	35,584 × 128 Matrix-2000[27]	TH Express-2	NUDT	National Supercomputer Center in Guangzhou China	2013	Linux (Kylin)
7▲ (new)	44.120 70.980	JUWELS (booster module)[28]	BullSequana XH2000	1,872 × 24 AMD EPYC 7402 @2.8 GHz	3,744 × 108 Ampere A100	Mellanox HDR Infiniband	ATOS	Forschungszentrum Jülich Germany	2020	Linux (CentOS)
8 ▼(6)	35.450 51.721	HPC5	Dell	3,640 × 24 Xeon Gold 6252 @2.1 GHz	7,280 × 80 Tesla V100	Mellanox HDR Infiniband	Dell EMC	Eni Italy	2020	Linux (CentOS)
9 ▼ (8)	23.516 38.746	Frontera	Dell C6420	16,016 × 28 Xeon Platinum 8280 @2.7 GHz (subsystems with e.g. POWER9 CPUs and Nvidia GPUs were added after official benchmarking[10])	0	InfiniBand HDR	Dell EMC	Texas Advanced Computing Center United States	2019	Linux (CentOS)
10▲(new)	22.400 55.424	DAMMAM-7	CRAY CS-Storm	1,978 × Xeon Gold 6248 @2.5 GHz	7,912 × 80 Tesla V100	InfiniBand HDR 100	Cray	Saudi Aramco Saudi Arabia	2020	Linux (RHEL)

QUANTUM PROCESSORS?





Not really

GOALS OF BENCHMARKING

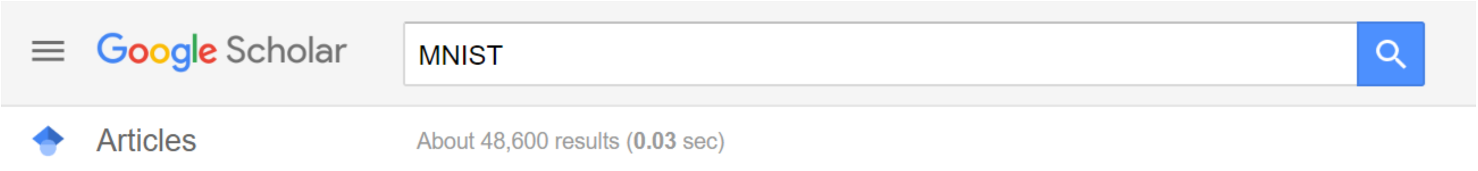
Help user to choose

Help developers

Address practical problems

Software benchmarking

MNIST: DATABASE OF HANDWRITTEN DIGITS

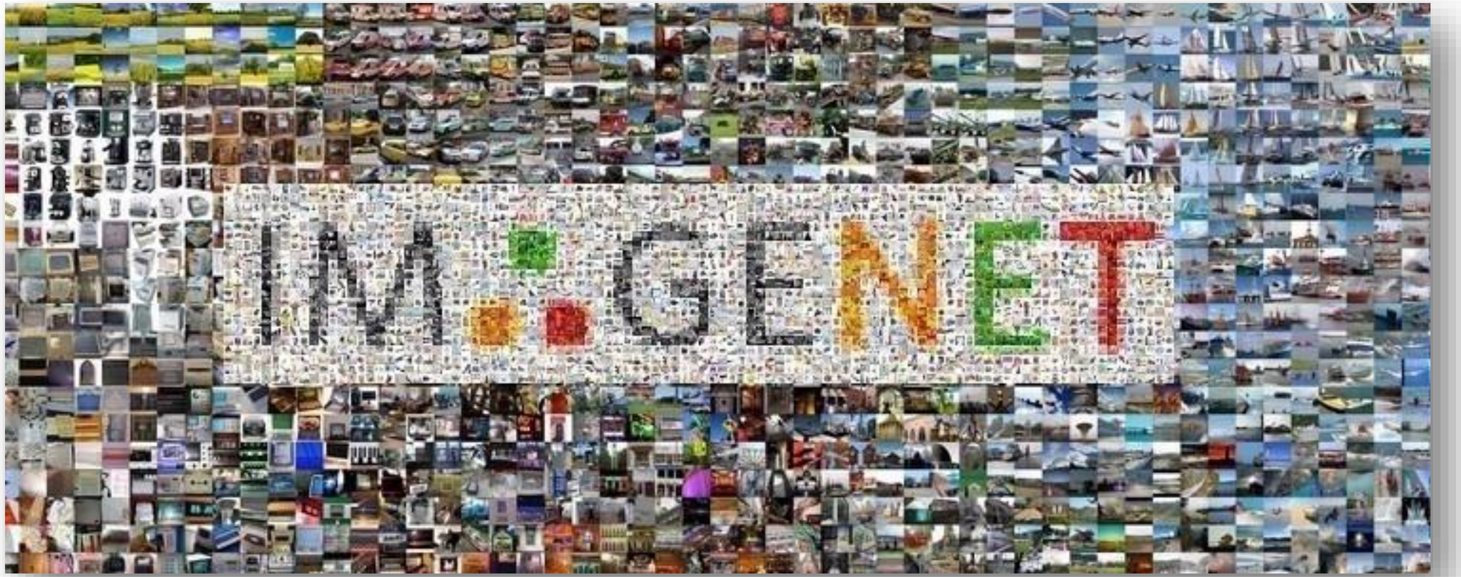


60,000 training set
10,000 testing set

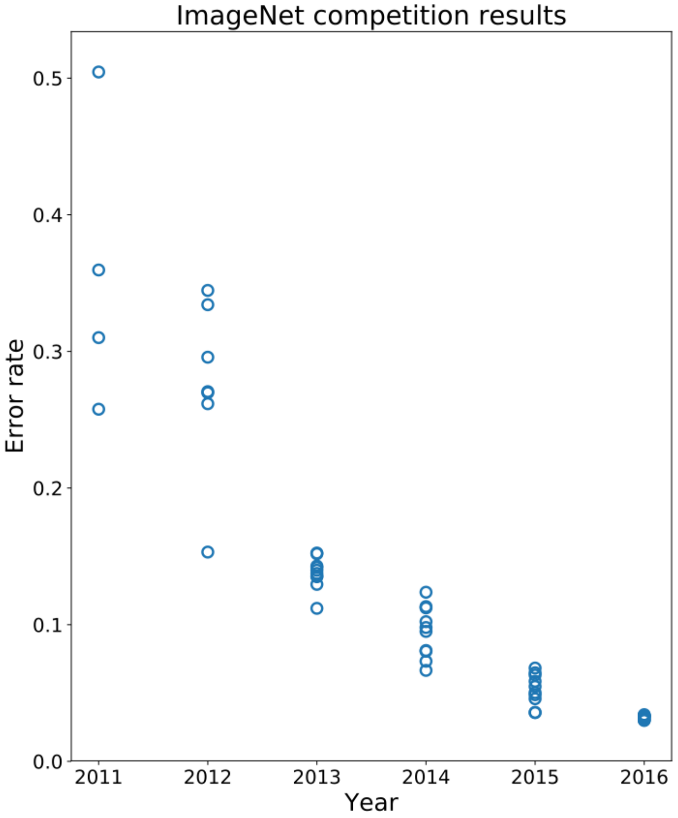
https://en.wikipedia.org/wiki/MNIST_database

Type	Classifier	Distortion	Preprocessing	Error rate (%)
Convolutional neural network	Committee of 20 CNNS with Squeeze-and-Excitation Networks[32]	None	Data augmentation	0.17[33]
Random Multimodel Deep Learning (RMDL)	10 NN-10 RNN - 10 CNN	None	None	0.18[22]
Convolutional neural network	Committee of 5 CNNs, 6-layer 784-50-100-500-1000-10-10	None	Expansion of the training data	0.21[19][20]
Convolutional neural network	Committee of 35 CNNs, 1-20-P-40-P-150-10	Elastic distortions	Width normalizations	0.23[12]
Convolutional neural network (CNN)	13-layer 64-128(5x)-256(3x)-512-2048-256-256-10	None	None	0.25[17]
Convolutional neural network	6-layer 784-50-100-500-1000-10-10	None	Expansion of the training data	0.27[31]
Convolutional neural network (CNN)	6-layer 784-40-80-500-1000-2000-10	None	Expansion of the training data	0.31[30]

IMAGENET:
DATABASE OF ANNOTATED IMAGES



> 14,000,000 images
> 20,000 categories



MORE MACHINE-LEARNING DATABASES

https://en.wikipedia.org/wiki/List_of_datasets_for_machine-learning_research

Contents [hide]

> 300 date-sets from 10 categories

BENCHMARKING PSEUDO-RANDOM NUMBER GENERATORS

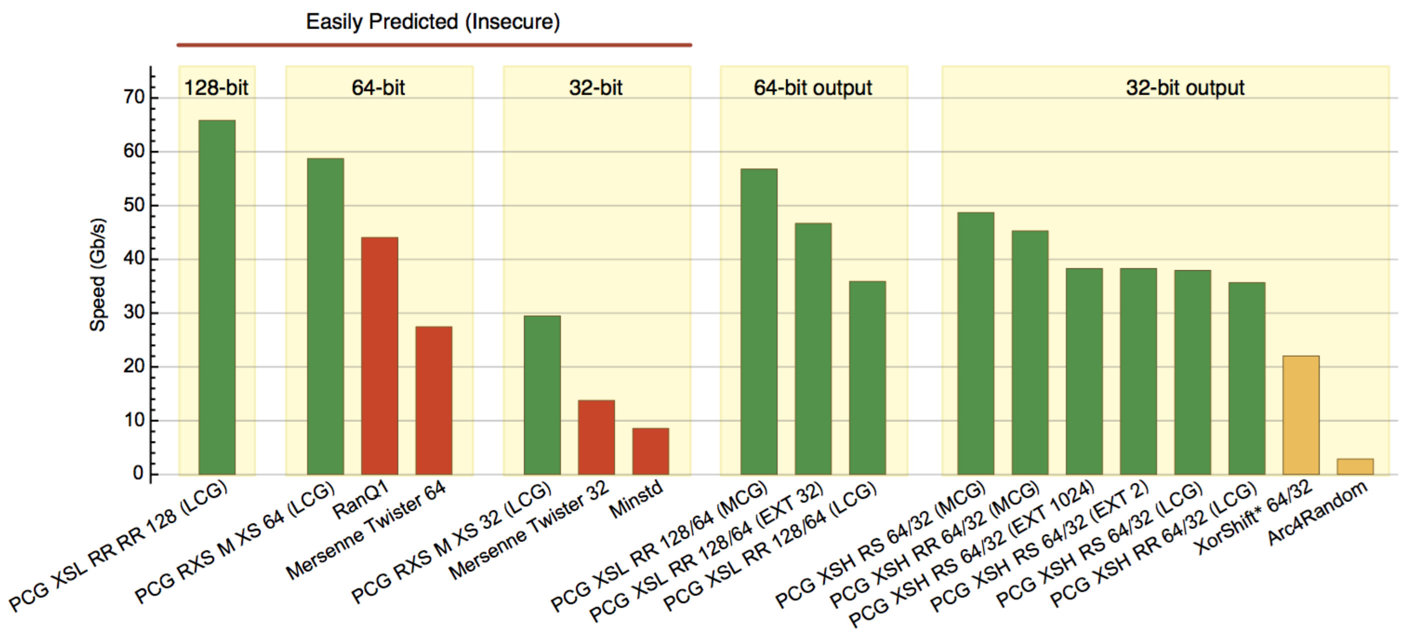
Random Bit Generation RBG

Guide to the Statistical Tests

A total of fifteen statistical tests were developed, implemented and evaluated. The following describes each of the tests.

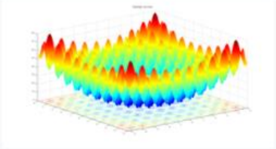
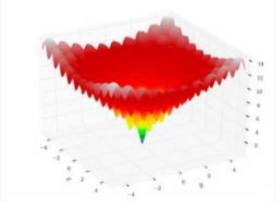
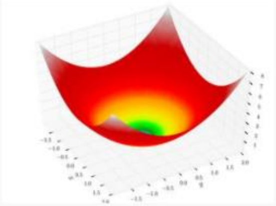
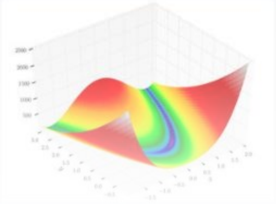
+ expand all

- Frequency (Monobits) Test
- Test For Frequency Within A Block
- Runs Test
- Test For The Longest Run Of Ones In A Block
- Random Binary Matrix Rank Test
- Discrete Fourier Transform (Spectral) Test
- Non-Overlapping (Aperiodic) Template Matching Test
- Overlapping (Periodic) Template Matching Test
- Maurer's Universal Statistical Test
- Linear Complexity Test
- Serial Test
- Approximate Entropy Test
- Cumulative Sum (Cusum) Test
- Random Excursions Test
- Random Excursions Variant Test



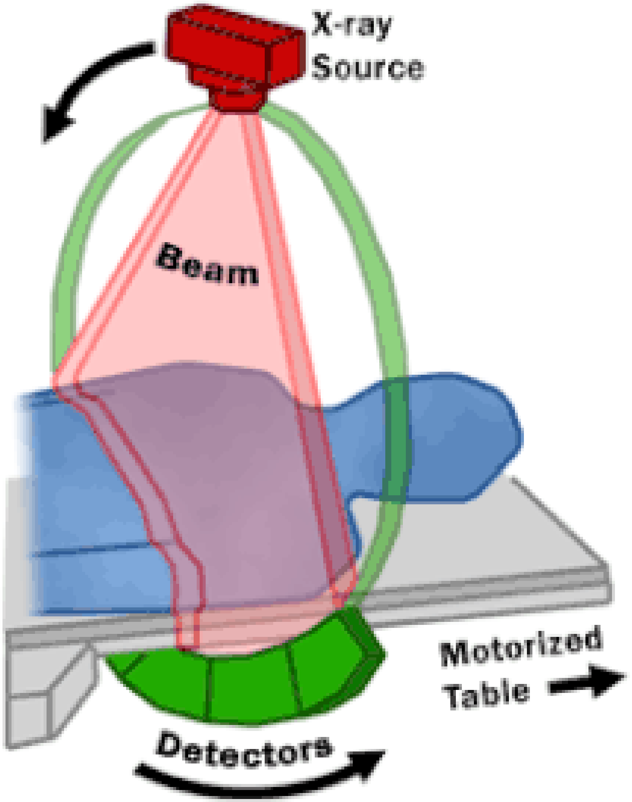
BENCHMARKING NUMERICAL OPTIMIZATION

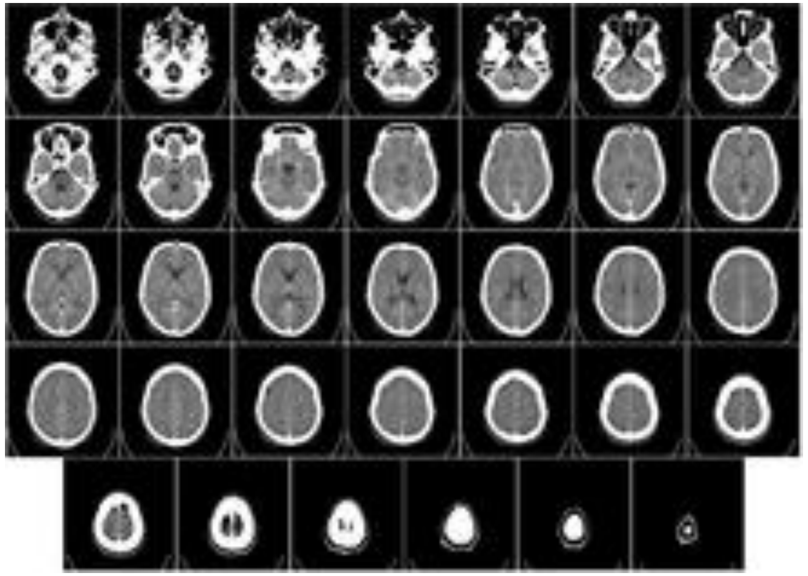
https://en.wikipedia.org/wiki/Test_functions_for_optimization

Name	Plot	Formula	Global minimum	Search domain
Rastrigin function		$f(\mathbf{x}) = An + \sum_{i=1}^{n} \left[x_i^2 - A\cos(2\pi x_i)\right]$ where: $A = 10$	$f(0,\dots,0) = 0$	$-5.12 \le x_i \le 5.12$
Ackley function		$f(x,y) = -20\exp\left[-0.2\sqrt{0.5\left(x^2+y^2\right)}\right] - \exp\left[0.5\left(\cos 2\pi x + \cos 2\pi y\right)\right] + e + 20$	$f(0,0) = 0$	$-5 \le x,y \le 5$
Sphere function		$f(\boldsymbol{x}) = \sum_{i=1}^{n} x_i^2$	$f(x_1,\dots,x_n) = f(0,\dots,0) = 0$	$-\infty \le x_i \le \infty$, $1 \le i \le n$
Rosenbrock function		$f(\boldsymbol{x}) = \sum_{i=1}^{n-1} \left[100\left(x_{i+1} - x_i^2\right)^2 + \left(1 - x_i\right)^2\right]$	$\text{Min} = \begin{cases} n = 2 & \rightarrow \quad f(1,1) = 0, \\ n = 3 & \rightarrow \quad f(1,1,1) = 0, \\ n > 3 & \rightarrow \quad f(\underbrace{1,\dots,1}_{n \text{ times}}) = 0 \end{cases}$	$-\infty \le x_i \le \infty$, $1 \le i \le n$

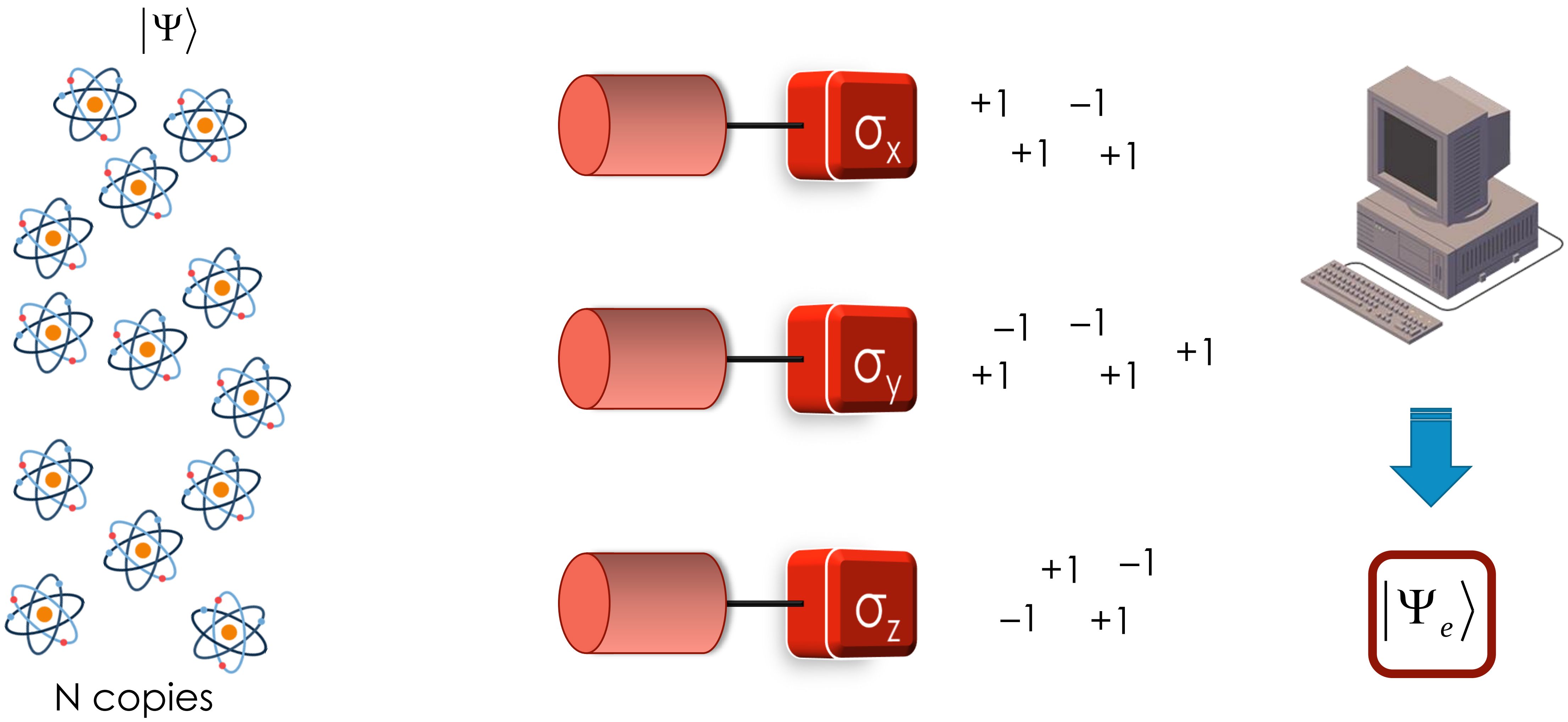
Quantum Tomography Benchmarking

COMPUTER TOMOGRAPHY

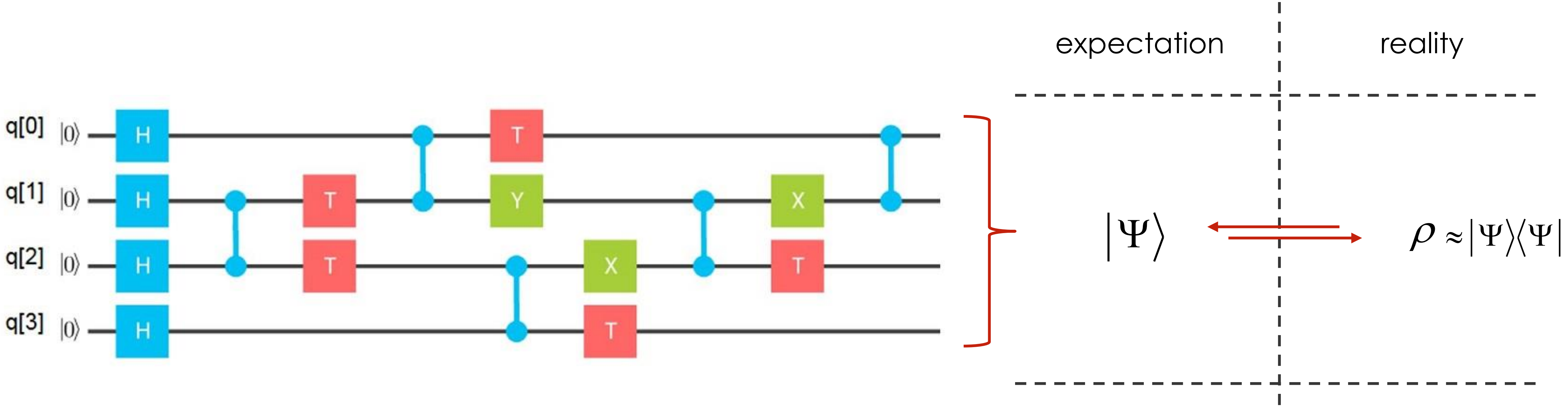




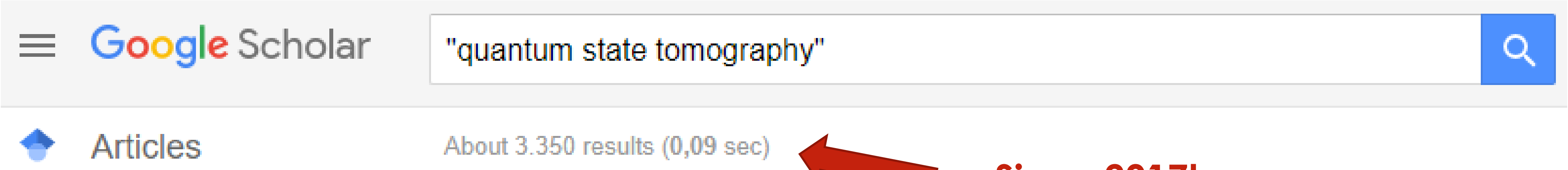
QUANTUM TOMOGRAPHY



QUANTUM TOMOGRAPHY AS A DEBUG TOOL



HOW TO IMPLEMENT QT?



Since 2017!
> 2 articles/day

Difficulties in comparison and choice

- ❑ Lack of important details
- ❑ Incredibly difficult to implement unknown methods
  - Difficult to verify implementations
- ❑ Strongly limited number of examples
- ❑ Different accuracy measures
- ❑ Different tests
- ❑ Publication bias

A LIFE STORY

New Journal of Physics
The open access journal at the forefront of physics

PAPER • OPEN ACCESS

Quantum tomography via compressed sensing: error bounds, sample complexity and efficient estimators

Steven T Flammia[5,1], David Gross[2], Yi-Kai Liu[3] and Jens Eisert[4]

Published 27 September 2012 •

New Journal of Physics, Volume 14, September 2012

Citation Steven T Flammia *et al* 2012 *New J. Phys.* **14** 095022

OTHER EXAMPLES

Superfast maximum-likelihood reconstruction for quantum tomography

Jiangwei Shang, Zhengyun Zhang, and Hui Khoon Ng
Phys. Rev. A **95**, 062336 – Published 27 June 2017

No details on protocol and reconstruction accuracy

Atoms, Molecules, Optics | Published: 22 July 2009

Unified statistical method for reconstructing quantum states by purification

Yu. I. Bogdanov

Journal of Experimental and Theoretical Physics **108**, 928–935(2009) | Cite this article

66 Accesses | **23** Citations | Metrics

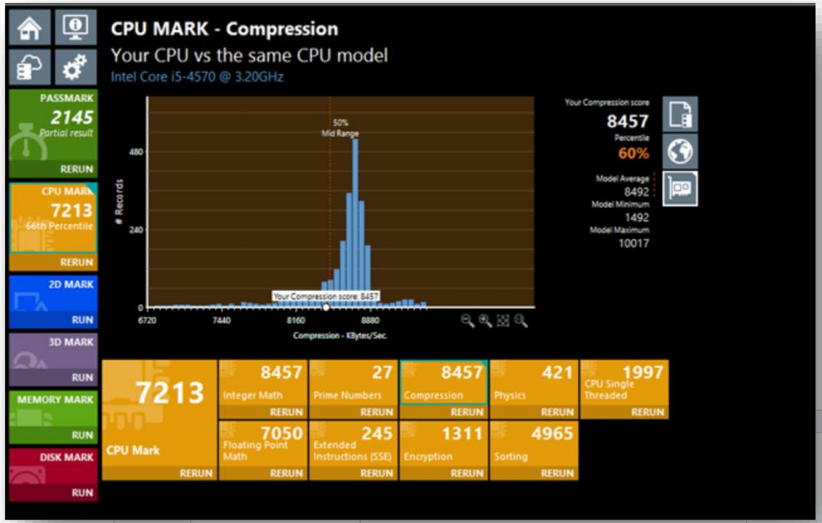
A single example of a mixed state

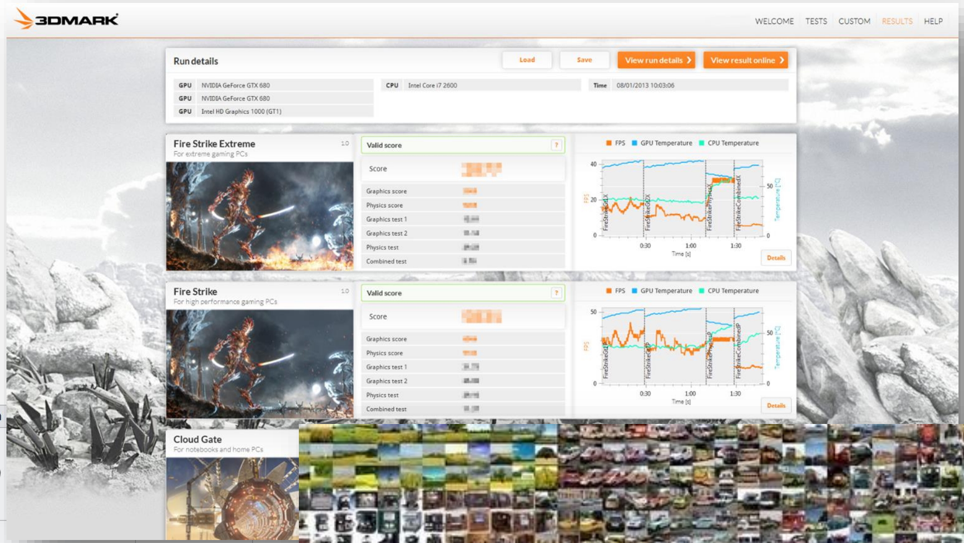
Adaptive quantum tomography of high-dimensional bipartite systems

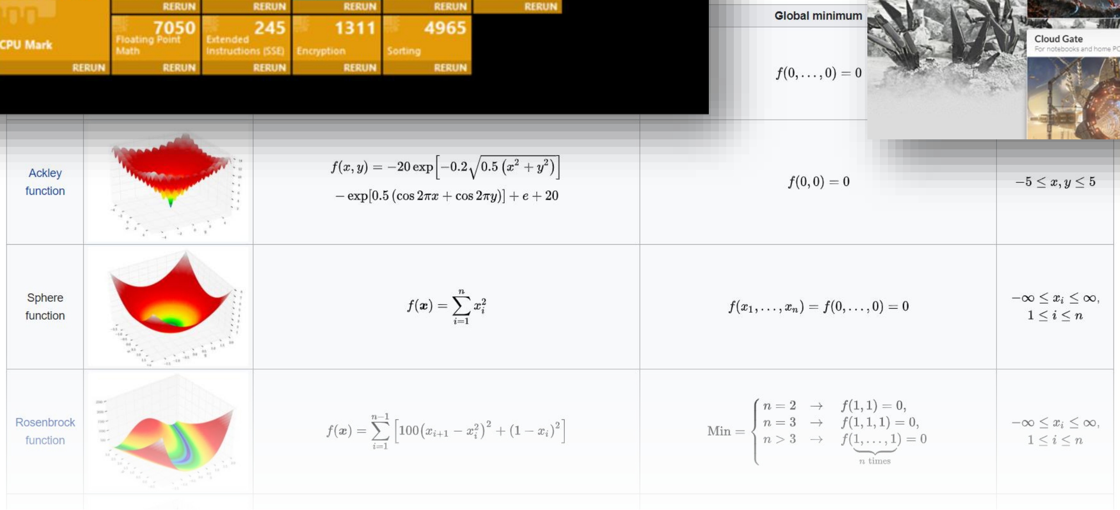
G. I. Struchalin, E. V. Kovlakov, S. S. Straupe, and S. P. Kulik
Phys. Rev. A **98**, 032330 – Published 26 September 2018

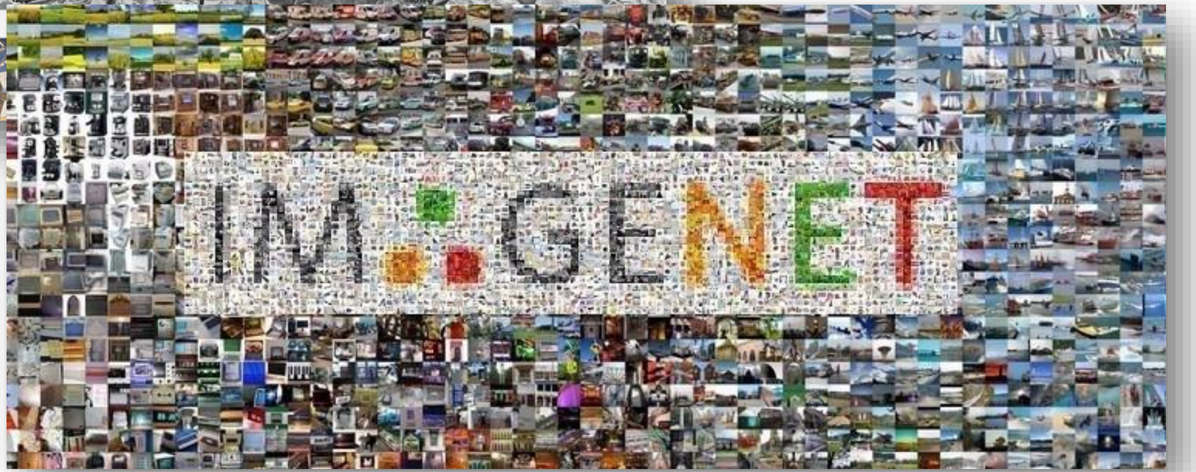
Only random high-dimensional bipartite states

RECALL THE PRACTICE FROM OTHER FIELDS









Quantum Tomography Benchmarking

QTB

Goal

Create a convenient universal platform for analyzing and comparing quantum tomography methods

Objectives

Develop <u>useful</u> comparison **metrics**

Design a **test suite** close to experimental problems

Develop easy-to-use **software**

Basic principles

Simulation based analysis

Tests and methods are completely independent

METRICS

What	Why
Fidelity	Main metric defining the tomographic accuracy
Total sample size	Large sample size require longer experiment
Number of bases	Each basis change require experimental setup re-configuration
Protocol computation time	Adaptive methods take time to calculate next measurement
State computation time	Non-effective method may take too much time
Efficiency	Defines the closeness to an perfect method
Outliers ratio	Outliers may spoil some experiments
Factorized measurements	Non-factorized are more noisy and hard to implement

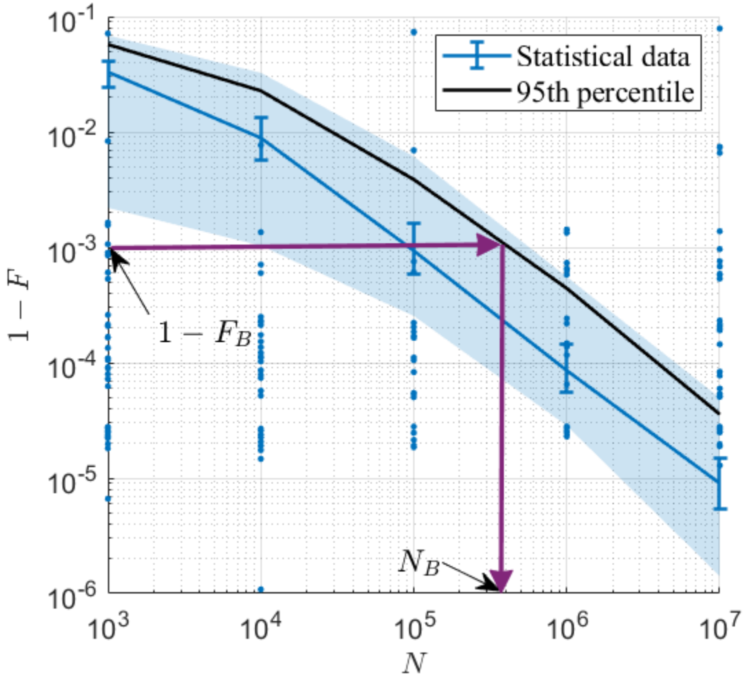
METRICS DERIVATION

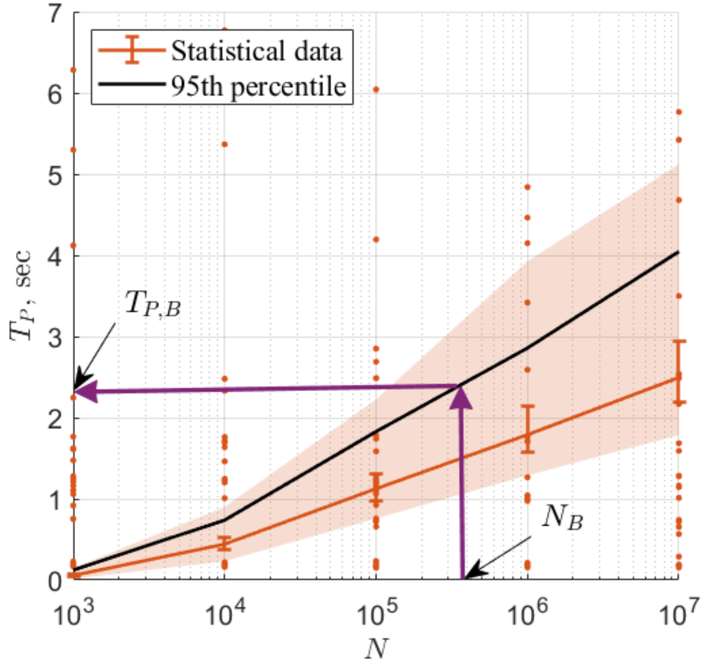
Common approach

« I have the sample size N. What fidelity can I get? »

Our approach

« What resources do I need to get the required fidelity? »





99,9% fidelity with 95% probability

BENCHMARK TESTS

✓ Random pure states

Common QT problem

$$\rho = |\psi\rangle\langle\psi|,\ |\psi\rangle = U|0\rangle,\ U \sim \mathrm{Haar}(d)$$

✓ Random mixed states by partial tracing

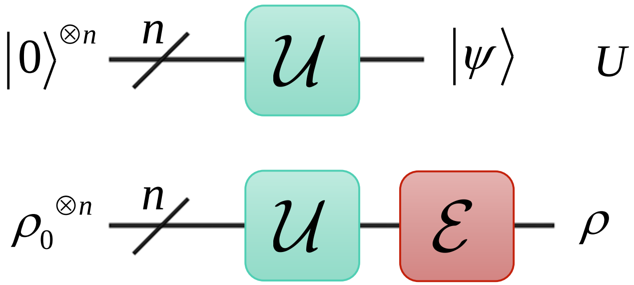
When analyzing a subsystem

$$\rho = \mathrm{Tr}_A\left(|\psi\rangle\langle\psi|\right),\ |\psi\rangle = U|0\rangle,\ U \sim \mathrm{Haar}(d_S \cdot d_A)$$

$d_A = 2 \qquad d_A = d_S$

✓ Random noisy preparation

Induced by preparation procedure



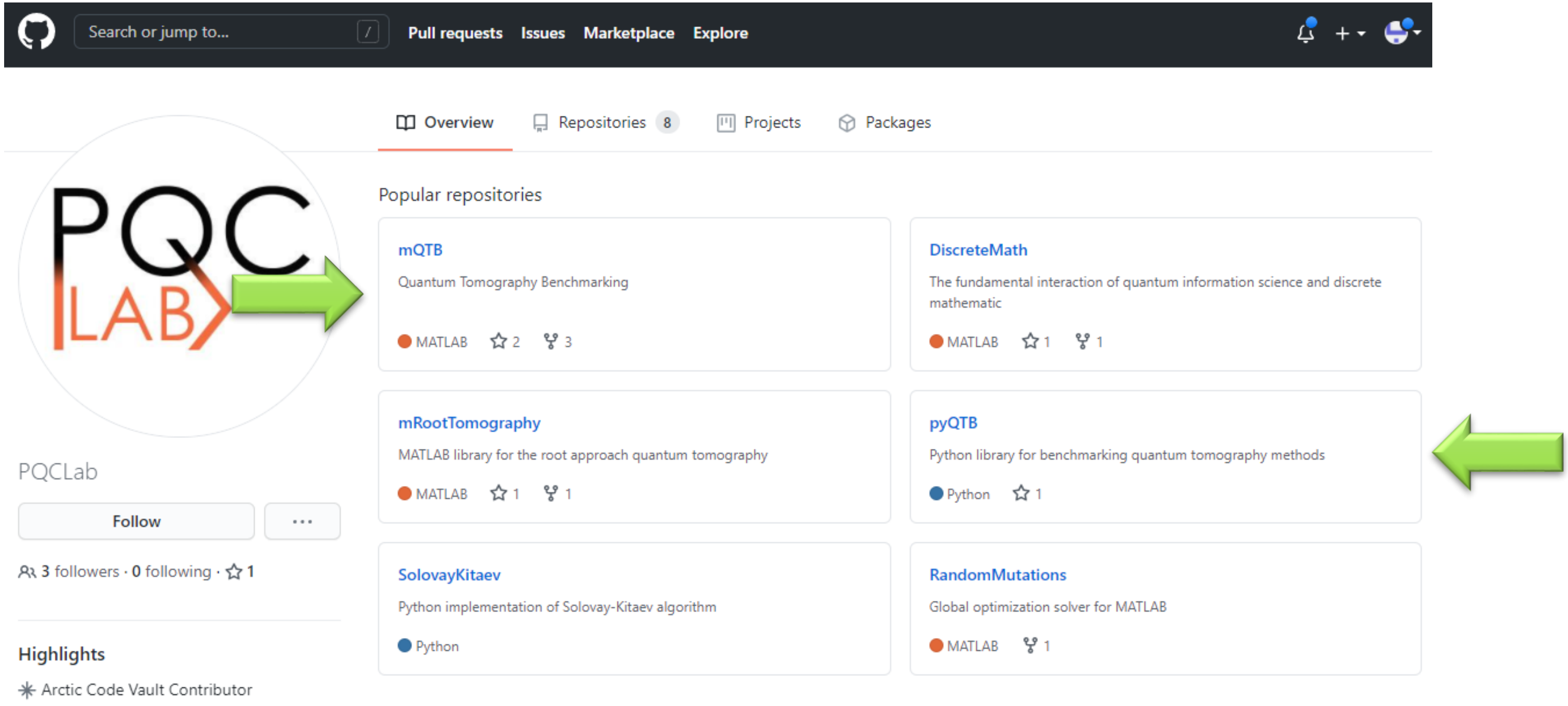
$U \sim \mathrm{Haar}(d)$

$$\rho_0 = (1-e_0)|0\rangle\langle 0| + e_0|1\rangle\langle 1| \qquad e_0 \sim \mathrm{unif}(0, 0.05)$$

$$\mathcal{E}(\rho) = (1-p)\rho + pI/d \qquad p \sim \mathrm{unif}(0, 0.01)$$

SOFTWARE

https://github.com/PQCLab



BASIC FEATURES

Easy-to-use

Random numbers synchronization

Parallelization (Python only)

Close to a real experiment

protocol handler

estimator handler

```
dim = [2,2];
result = qtb_analyze(proto_fmub(dim), est_ppi(), dim, 'rps');
qtb_report(result, 'rps', 'export', 'report.xlsx');
```

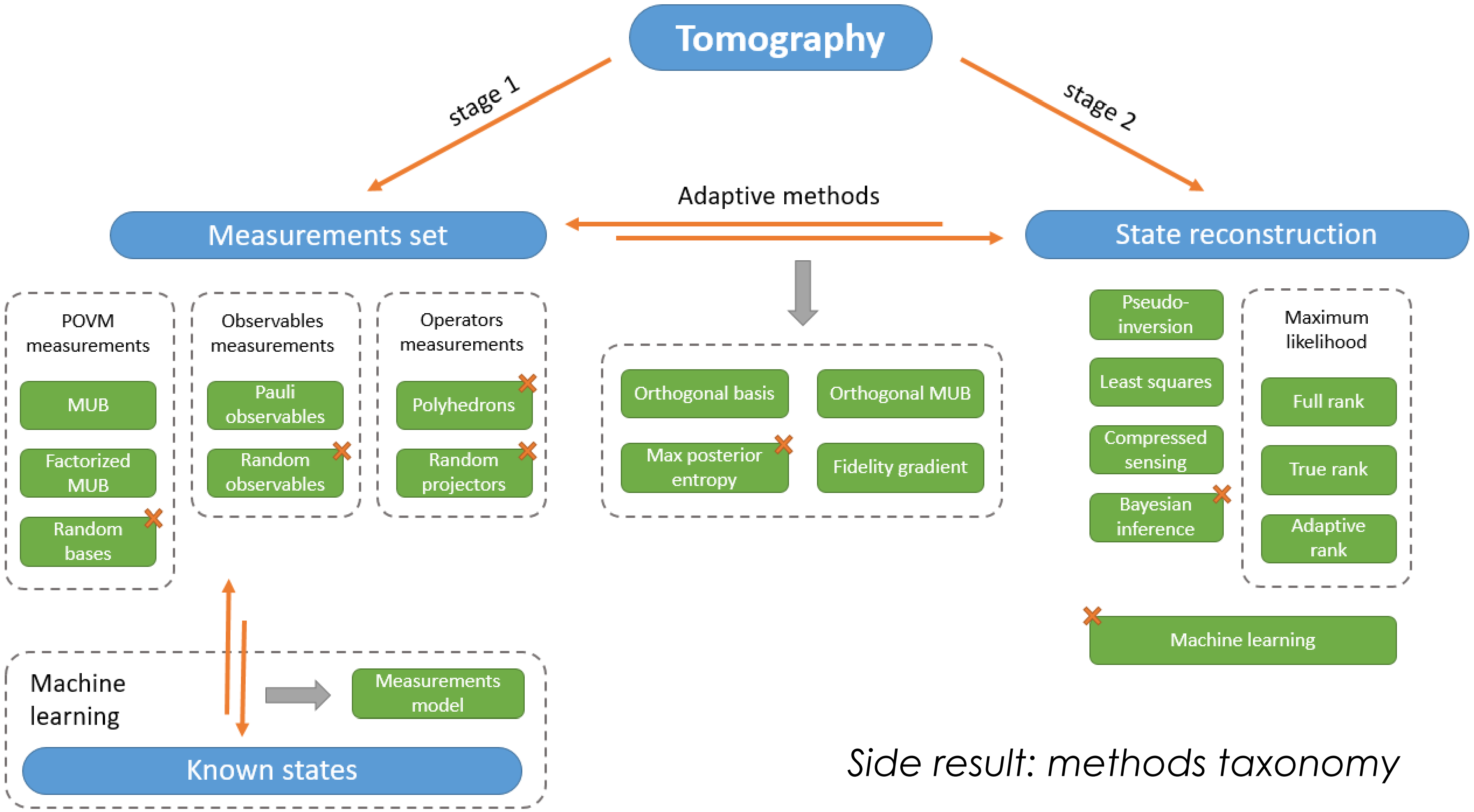
Report generation (MATLAB only)

Custom tests (not released yet)

Same interface for MATLAB and Python

MATLAB ↔ Python results migration

METHODS



Side result: methods taxonomy

APPROBATION

Supplementary material I. Methods analysis results

Quantum tomography benchmarking

Bantysh B. I., Chernyavskiy A. Yu., Bogdanov Yu. I.

The current document summarizes the results of using the quantum tomography (QT) benchmarking software for analyzing different QT methods. For each method we also provide a basic information: measurement protocol, quantum state estimator, data processing algorithm and computing machine. The sign "*" in tables means that the value was obtained by linear extrapolation of the dependence of $\log[1 - F]_{95}$ on $\log N$.

Contents

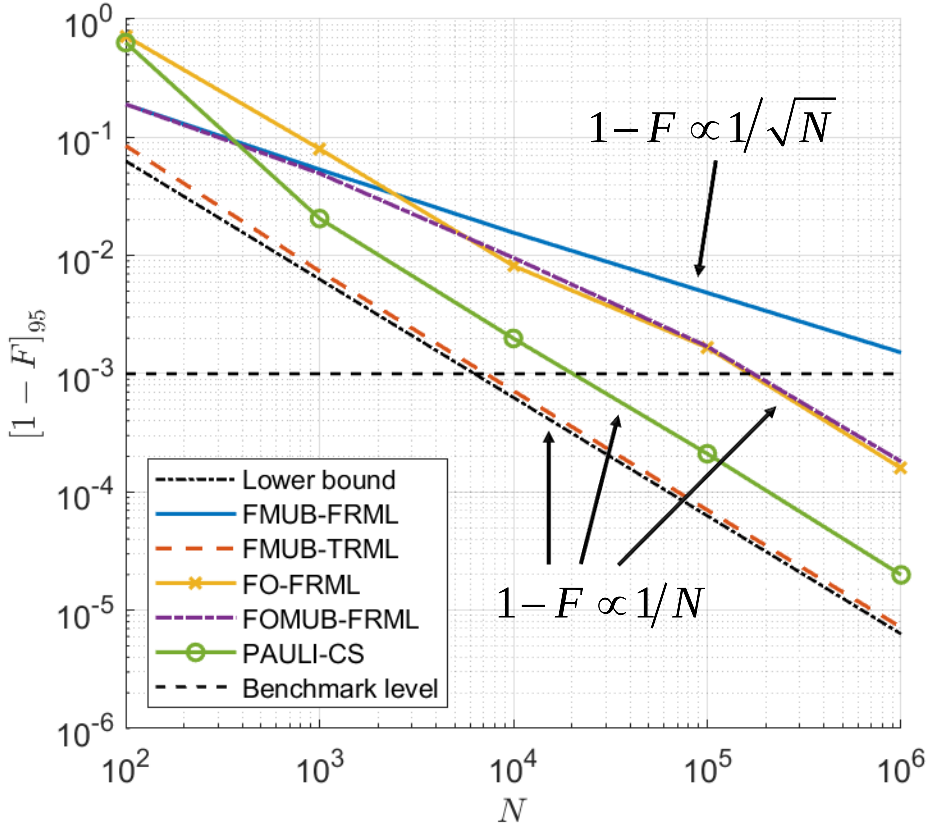
Bantysh B. I., Chernyavskiy A. Y., Bogdanov Y. I. Quantum tomography benchmarking //arXiv preprint arXiv:2012.15656. – 2020 (under review in Quantum Information Processing)

Special thanks to Dr. G.I. Struchalin for help in carrying out the computations

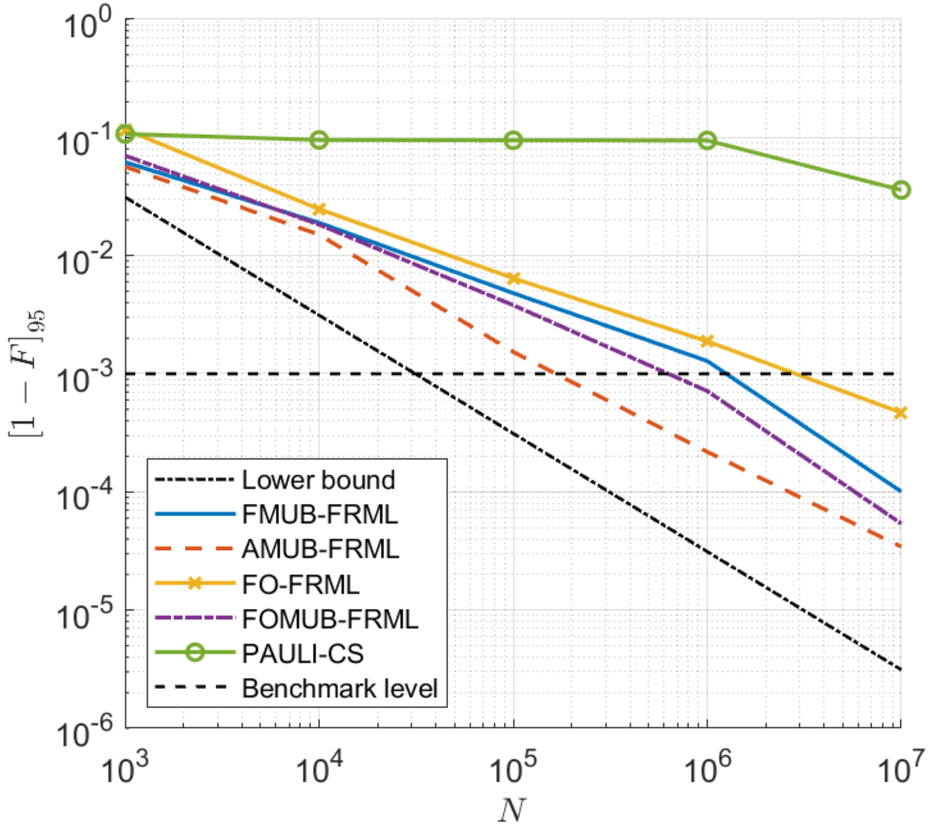
12 methods
1, 2, 3 qubits
127 tables

VISUAL REPRESENTATION: TWO QUBITS CASE

Random pure states test



Random noisy preparation test



QUANTITATIVE REPRESENTATION: TWO QUBITS CASE

Table 1 Comparison of benchmarks of tomography methods for two-qubit pure states (reaching benchmark fidelity $F_B = 99.9\%$ with 95% probability)

	N_B	M_{95}^B	$T_{P,95}^B$, sec	$T_{E,95}^B$, sec	η_B	O_B	FM
Lower bound	6 296	–	–	–	1	–	–
FMUB-TRML	7 205	9	0.0014	0.0081	0.9	0.0098	Y
FMUB-ARML	7 798	9	0.0014	0.013	0.57	0.022	Y
PAULI-CS	20 239	15	0.0015	0.61	0.31	0.0071	Y
AMUB-FRML*	23 472	98	2.9	0.31	0.35	0.0026	N
FO-FRML*	164 734	153	170	0.14	0.062	0.048	Y
FOMUB-FRML*	172 765	172	15	1.1	0.073	0.0078	Y
SGQT*	**1 048 989	–	–	–	–	–	N
FMUB-CS	**1 123 593	–	–	–	–	–	Y
FMUB-FRML	**1 424 621	–	–	–	–	–	Y
MUB-FRML	**1 638 550	–	–	–	–	–	N
FMUB-FRLS	**1 763 979	–	–	–	–	–	Y
FMUB-PPI	**1 811 355	–	–	–	–	–	Y

* Adaptive method

** The value is obtained by linear extrapolation of the dependence of $\log[1-F]_{95}$ on $\log N$

QUANTITATIVE REPRESENTATION: TWO QUBITS CASE

Table 2 Comparison of benchmark parameters of tomography methods for two-qubit states with noisy preparation (reaching benchmark fidelity $F_B = 99.9\%$ with 95% probability)

	N_B	M_{95}^B	$T_{P,95}^B$, sec	$T_{E,95}^B$, sec	η_B	O_B	FM
Lower bound	31 245	–	–	–	1	–	–
AMUB-FRML*	165 667	161	5.2	0.2	0.28	0.0083	N
FOMUB-FRML*	629 755	218	36	2.1	0.095	0.036	Y
MUB-FRML	1 016 042	5	0.00067	0.022	0.072	0.056	N
FMUB-FRML	1 251 861	9	0.00051	0.18	0.06	0.039	Y
FMUB-TRML	1 251 861	9	0.00051	0.18	0.06	0.039	Y
FMUB-FRLS	1 382 130	9	0.0011	0.66	0.05	0.038	Y
FMUB-PPI	1 400 187	9	0.00061	0.00057	0.049	0.038	Y
FMUB-CS	2 234 214	9	0.00091	0.49	0.027	0.014	Y
FO-FRML*	2 851 554	240	301	3.5	0.026	0.014	Y
FMUB-ARML	6 238 296	9	0.0012	0.47	0.017	0.13	Y
PAULI-CS	**39 664 389	–	–	–	–	–	Y

* Adaptive method
** The value is obtained by linear extrapolation of the dependence of $\log[1-F]_{95}$ on $\log N$

CONCLUSIONS

- A methodology for benchmarking quantum tomography methods was implemented
  - A set of reference (benchmarking) parameters has been developed
  - Tests that are close to experiments have been developed
  - The corresponding universal software has been developed
- Approbation was carried out on 12 methods of quantum tomography
- A taxonomy of quantum tomography methods has been developed

We encourage all scientists who develop and use quantum tomography methods to participate in the development!

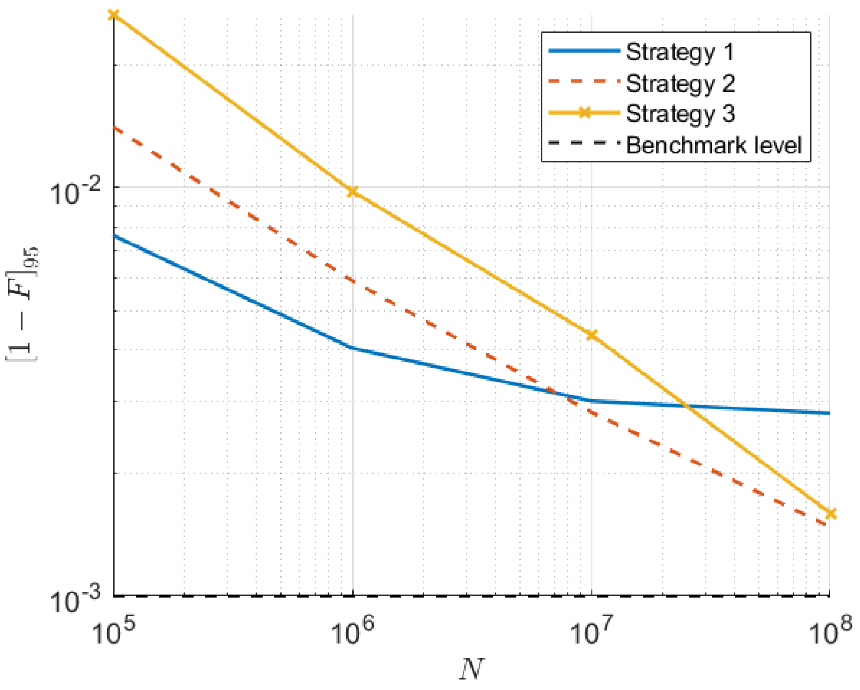
FUTURE WORK

- Support methods with training
- Support prior information use
- Tests with systematic measurement errors
- Quantum process tomography

SIDE RESULT: TEST BENCH FOR QT METHODS

Task: compare different measurement strategies in the presence of instrumental noise

```
qudit_test = qtb_tests.create_test(d, ...
    @() statefun(d, depol), ...
    @(dm, meas) measfun(dm, meas, d, pm, depol), ...
    'code', 'qudit', ...
    'nsample', round(10.^[5,6,7,8]), ...
    'nexp', 200, ...
    'rank', d);
```



	NSamples
Strategy 3	'*164 259 424'
Strategy 2	'*226 686 292'
Strategy 1	'*1.052 531 715 800 418e+17'

Thank you for your attention!

https://en.wikipedia.org/wiki/List_of_quantum_tomography_methods

Protocol	Estimator	Sample size	Measurement number	Efficiency (%)
Factorized MUB	True rank maximum likelihood	7 205	9	90
Factorized MUB	Adaprive rank maximum likelihood	7 798	9	57
Pauli observables	Compressed sensing	20 239	15	31
Adaptive MUB	Full rank maximum likelihood	23 472	98	35
Factorized orthogonal	Full rank maximum likelihood	164 734	153	6.2

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METRICS DETAILS

$$F = \left(\mathrm{Tr}\sqrt{\sqrt{\rho}\sigma\sqrt{\rho}}\right)^2$$ – fidelity

$$\eta = \frac{\langle 1-F\rangle_{\min}}{\langle 1-F\rangle} = \frac{\left[\left(2d-r\right)r-1\right]^2}{4N\left(d-1\right)\langle 1-F\rangle}$$ – efficiency

$$MC = -\operatorname*{med}_{F_- < Q_2 < F_+}\left[\frac{\left(F_+ - Q_2\right) - \left(Q_2 - F_-\right)}{F_+ - F_-}\right]$$

$$1 - F \in \left[Q_1 - 1.5e^{-4MC} IQR;\quad Q_3 + 1.5e^{3MC} IQR\right]$$ – outliers